# Recommendation

## A STUDY OF SOME FIXEDPOINT THEOREMS IN THE CONTEXT OF SOFT SETS & ROUGH SETS AND THEIR APPLICATIONS

Functional analysis is considered a major branch of pure mathematics. It has enormous applications in the field of applied mathematics and other branches of science.

Set theory, as a foundational and structural concept in mathematics has witnessed multiple paradigm shifts in the journey of its evolution. Thanks to the increasing inclination of the scientific world towards embracing the uncertainties infused in real-life problems which exposed the limitations of classical set theory. Zadeh in his seminal paper [119] turned heads by extending the concept of belongingness from a dichotomy to a degree. This paved the way to extend the classical set theory to the fuzzy dimension. Molodtsov in 1999 found an entirely new realm to extend the theory of classical sets. In [68] he first presented the notion of a soft set which took into consideration more than one parameter to group elements of a given universal set. Now, unlike classical sets where sets are defined based on only one criterion, in soft sets we can use more than one defining criteria which are called parameters here. Hence a soft set is called a parameterized family of subsets of a given universe. To model it set theoretically, the collection was described as a rule of correspondence from the parameter set to the power set of the universe. This immediately caught the attention of researchers as a tool to expand the context of any problem by including more than one parameter or criteria. Soft set theory was then developed by Maji et al in [39] wherein they defined null soft set, absolute soft set, soft subset, soft union, soft intersection, soft complement and binary operations of 'AND', 'OR', 'NOT' on soft sets. Pei and Miao [85] refurbished the definitions of soft intersection and soft subsets in [6]. Some soft set operations were further modified by Ali et al. in [1] including the notion of relative complement. Parameterized reduction of soft sets was studied by Chen et al. in [18] in the line of attribute reduction of rough set theory and by Xie in [114] that connected it to attribute reduction in information system. The development of soft sets is systematically recorded in the work of [82]. The concept of functions in soft set theory was explored in different avenues by various scholars. Babitha and Sunil in [11], Wardowski in [111], Samanta and Das in [13] and Yazar et

al in [116], all had different versions of soft mappings or soft functions. As an example, Babitha and Sunil [11] introduced these in the line of Cartesian product and soft relations whereas Wardowski [111], who first defined soft elements, drew an analogy between soft and classical mappings. Continuity of soft mappings was discussed by Aras et al. in [2]. A landmark progress in this journey was the inception of soft topology by Shabir and Naz in [98] and Cagman et al. in [23], [24], albeit their approaches were strikingly different. While the former defined soft topology by imposing soft sets over an initial classical universe, the latter defined soft topology on the soft subsets of initial soft sets. Following the path of [98], Hussain and Ahmad in [51] studied soft topological spaces and Nazmul and Samanta studied neighborhood properties of these spaces in [35]. Das and Samanta[35-37] widened our understanding of the theory of soft sets by conjoining it with the real number system and defining soft real sets, soft real numbers and soft complex numbers and their properties, operations in details. Again this duo initiated a wholesome field of soft metric spaces by postulating the axioms of soft metric in [36] which were functions defined on soft points, whose idea was also proposed in this same work. Carrying this legacy ahead, Yazar et al in [116] defined soft contractions and studied soft metric spaces in details to the extent of establishing some important fixed point results in soft metric spaces.

But merely highlighting a theory without proving its applicability makes it unconvincing in the practical world. That is where soft set theory has carved a niche for itself in the hall of non-classical set theories. Molodtsov in his torchbearer paper of [68] discussed plenty applications of soft sets. Finding a better pay function, modeling a choice function in an efficient way in game theory and operations research, the internal and external regularization of the classical Pareto set using the soft Pareto function in stability regularization, all were touched upon by Molodtsov. Later in [69-70] Maji et al. used rough mathematics of Pawlak and demonstrated an application of soft set theory in decision making. Decision making is one such area where soft set theory proves its mettle due to being inclusive of more than one parameter. In [80] Nasef and Sayed illustrated the dilemma of a buyer while purchasing a car and modeled it using soft sets. Pei and Miao in [85] studied the connection between soft sets and information systems and showed that their studies can be unified as soft sets are but special information systems. In the field of functional analysis soft set theory has been successfully applied on fixed point theorems to find newer generalized results and soft fixed point theory has been studied rigorously in scholars recent times.

Several traditional methods and tools which are used in formulation of models and useful in computations and reasoning problems are classical and very precise in nature. However, many complicated tasks in the studies of engineering, social science, economics, bio science etc include ambiguity in their data.

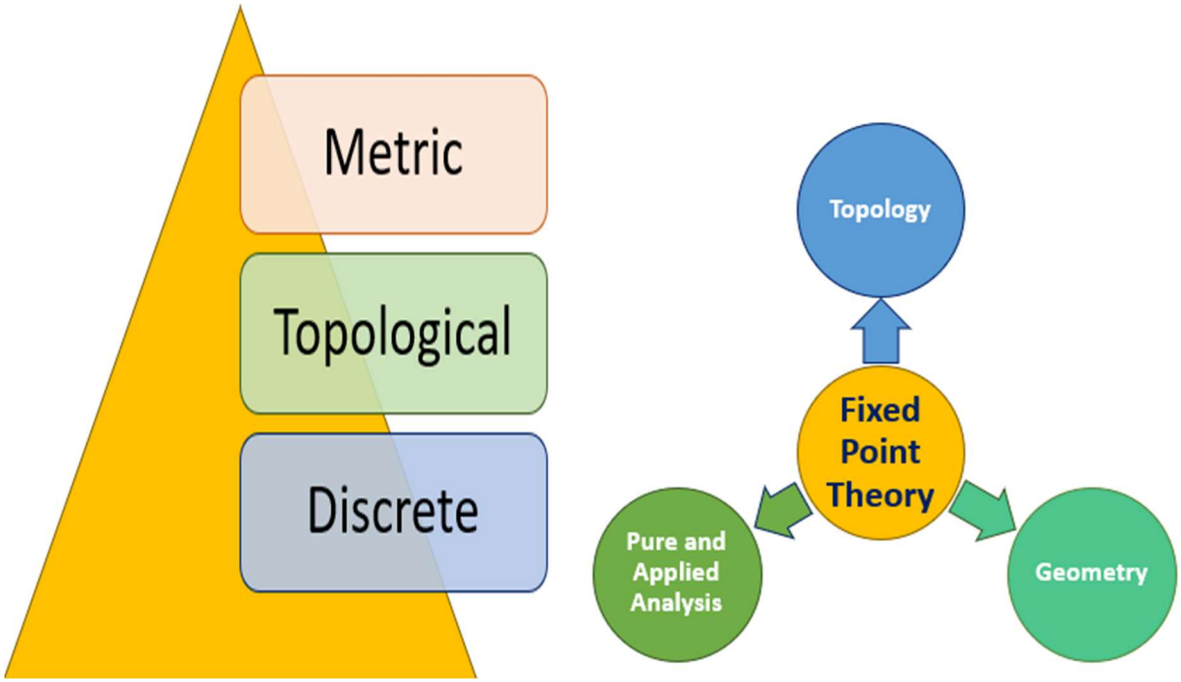
When the classical set approach failed to give solutions to problems that were based on unclear, undefined objects, the principal of Rough Set was formed. The first wide application of the Theory of Rough Set was seen in the world of Computers and Computer Science. Polish computer scientist Z. Pawlak [117]was the first to introduce the concept of approximation in Crisp or well-defined set-in terms of Upper or Lower Approximation for a given data set. The theory was precisely based on uncertainty in each data set.

Some times in information as well as material science, knowledge is regulated in a manner which has unspecific and inexplicit properties. In order to direct such affairs of the information control systems , the evidence theory, theory of fuzzy sets etc in $20^{th}$ century were developed. The approach based upon fuzzy sets was intitiated by L. Zadeh in the year 1965. In this regard, notion of Rough Sets as a different mathematical approach was proposed through Z. Pawlak [117]in 1982 in the "International Journal of Computer and information Sciences. Later the concept of Rough sets proposed for a very wide range of district applications such as discovery of knowledge , data mining , AI and cognitive science , in machine learning , approximation theory, pattern identifications etc.In 2006, Z. Pawlak and A. Skowron [118] , some extensions in rough set approach was presented with rough mereological approach and applications along approximate reasoning. Some application of it in Data Analysis and data mining, concurrent data modelling from given data tables etc. were discussed. The notion of Rough Fuzzy set and fuzzy rough sets was introduced in 1990 wherein the lower and upper approximations in fuzzy sets was applied to the approximate space given again by Pawlak. The concept of soft Rough fuzzy sets and Soft fuzzy Rough sets as a new model was established and properties of the same were discussed. The lower and upper approximations in this regard of fuzzy sets with related properties were surveyed. In 2001 H. X. Phu, the notion of Rough convergence was introduced in a normed linear space wherein discussion upon the outcomes of rough convergence and compression and dependency of rough convergence on simple convergence was given. Various studies has been done in various directions by many researchers using the notion given by H.X. Phu [52] discussed upon sequentially rough convergence as a generalization in normed linear

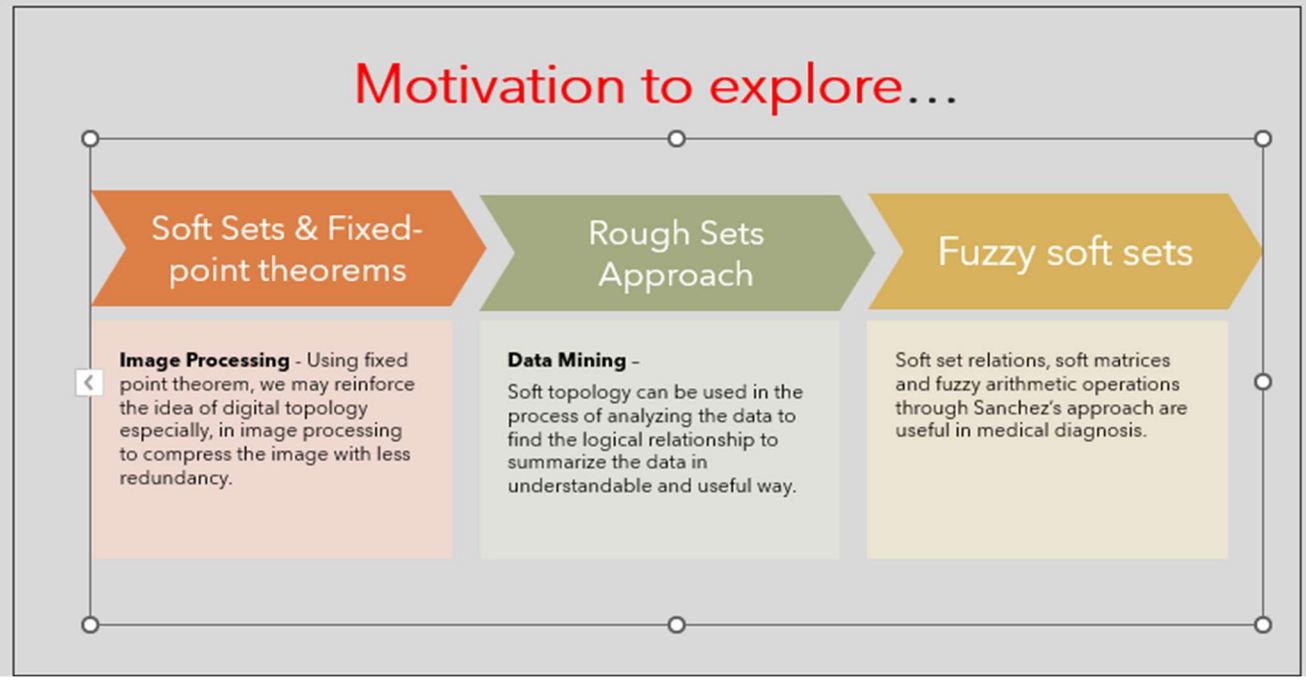
space with infinite dimension in [2003]. Afterword in [2018], using the idea of sequence rough convergence with cone metric space was given wherein several basic properties related to the concept were formed and studied. By P.K. Singh and S Tiwari [89] gave a survey on topological structures in rough set theory. In this survey, the significant work done in the area of crisp rough set theory and topology and their interdependencies during the past 20 years. Algebraic as well as logical aspects of topology on Rough sets are discussed in this survey. The new idea of roughness to vector space concepts was approached in [2019] by S.T. Almohammadi and C. Ozel [99]. In that article, mainly the idea of upper approximation was used to introduce rough vector space and proved some results in algebra. Addition to that topological space concept was merged with the given was new concepts of rough vector space. Whereas, on the other hand, the notion of Rough convergence in 2 – Normed space was brough. In [1996] R. Biswas introduced the notion of Rough metric space with a study on the concepts such as rough diameter , rough open ball , rough interior points, rough open set etc. In [1999] R. Biswas [90-92] defined rough Cauchy sequence completeness in rough metric space and the rough fixed point along with a Banach's type fixed point theorem in rough metric spaces and studied the said space in context to other topological spaces.

Fixed points are studied with great enthusiasm because of their wide range applications. Be it cosmology, digital image processing or differential and integral equations, fixed points are highly useful. Fixed points have mesmerized mathematicians because of their beautiful mathematical properties. With their dawn in $19^{th}$ century by works of Poincare [86], Cauchy (1884) and Picard, many mathematicians joined the league and carried on research in this direction. A popular method – called method of successive approximations – was suggested by Picard for proving existence theorems. Birkhoff and Kellogg gave the proof of classical existence theorem for function spaces in 1922 [8]. In the same year, Stefan Banach proposed the Banach Contraction Principle [10] which accelerated the research of functional analysis. This method was actually the result of geometric interpretation of Picard's method of successive approximations. The study of fixed points has been broadly divided into three approaches - Metric, Topological, and Discrete. The work of Cheema in [21] can be read for a holistic view of the basics of the subject. Some extensions of fixed-point results to soft metric were successfully done by Abbas et al in [3] wherein they illustrated their results with suitable examples. Yazar et

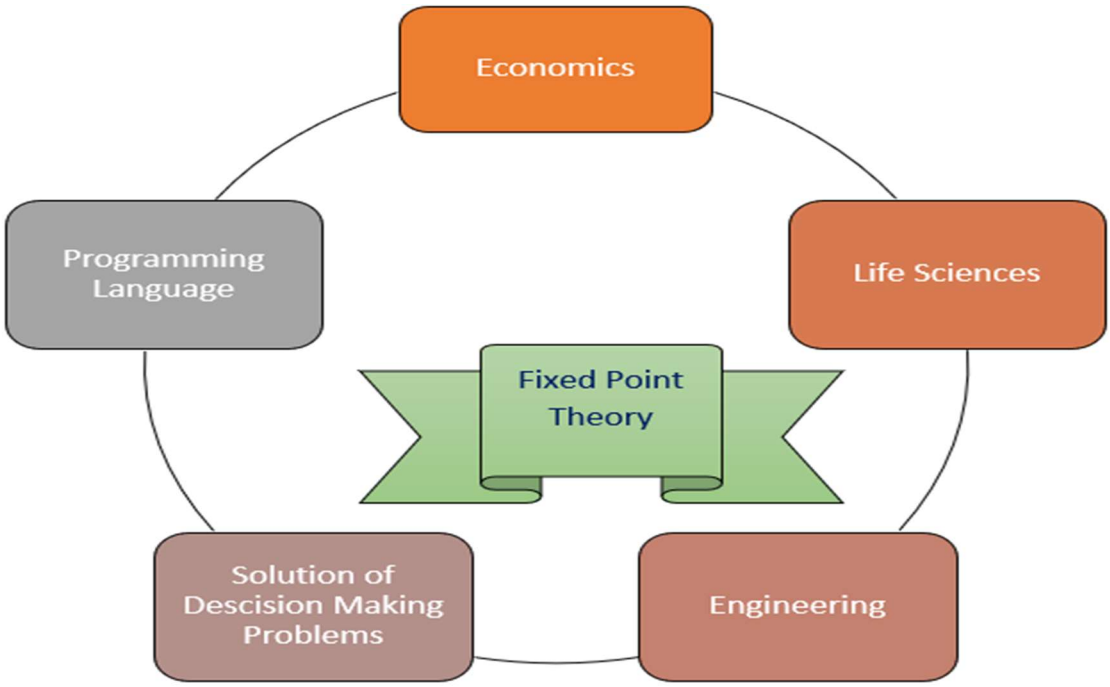
al. in [116] also proved some fixed-point results of soft contractive mappings in soft metric spaces. Researchers have also worked on fixed point theorems of variations of soft metric spaces Different approaches for fixed point theory are shown through figures bellow .



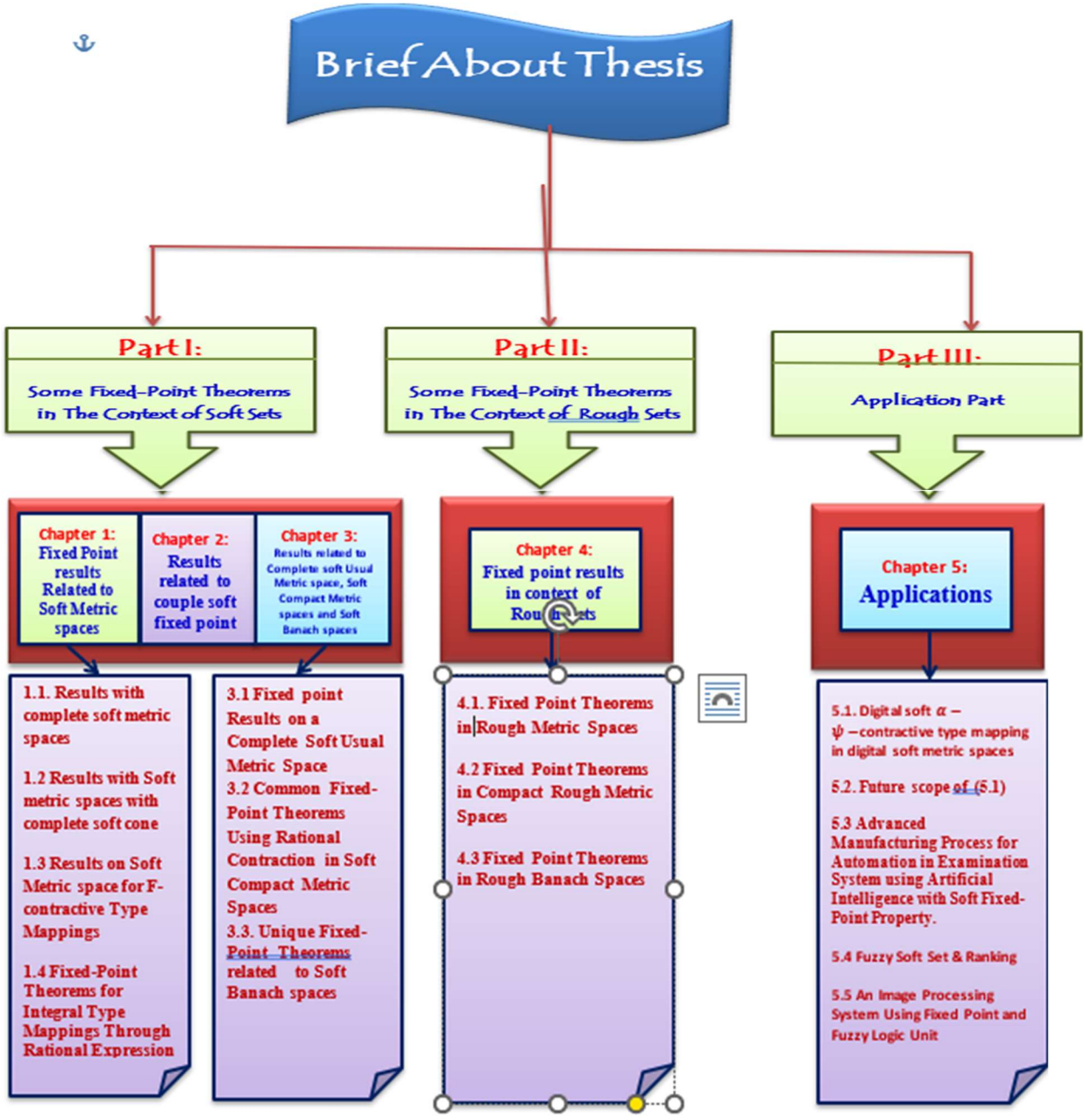

In the 19th century The study of fixed point theory was initiated by Poincare and in 20th century developed by many mathematicians like Brouwer, Schauder, Kakutani, Banach, Kannan and others. In 1912, Brouwer published his famous $\mathbb{F}ixed_{\mathbb{P}oint}$ theorem. The theorem states that, every continuous self-map of closed unit ball in n-Euclidean space possess a $\mathbb{F}ixed_{\mathbb{P}oint}$. Afterwards, in 1930 Schauder gave the first $\mathbb{F}ixed_{\mathbb{P}oint}$theorem in an infinite dimensional Banach space. In 1922 Banach proved a classical $\mathbb{F}ixed_{\mathbb{P}oint}$theorem which has many applications in the existence and uniqueness problems of differential equations and integral equations.


Motivation to explore…
Soft Sets & Fixed-point theorems
Rough Sets Approach
Fuzzy soft sets
Image Processing - Using fixed point theorem, we may reinforce the idea of digital topology especially, in image processing to compress the image with less redundancy.
Data Mining –
Soft topology can be used in the process of analyzing the data to find the logical relationship to summarize the data in understandable and useful way.
Soft set relations, soft matrices and fuzzy arithmetic operations through Sanchez's approach are useful in medical diagnosis.

## Applications of fixed-point theory



**BRIEF ABOUT THESIS:** The present work is mainly divided in three parts, whether the definitions and brief about soft sets, rough sets and fixed point are discussed in introduction and preliminaries section.


Brief About Thesis
Part I:
Some Fixed-Point Theorems in The Context of Soft Sets
Part II:
Some Fixed-Point Theorems in The Context of Rough Sets
Part III:
Application Part
Chapter 1: Fixed Point results Related to Soft Metric spaces
Chapter 2: Results related to couple soft fixed point
Chapter 3: Results related to Complete soft Usual Metric space, Soft Compact Metric spaces and Soft Banach spaces
Chapter 4: Fixed point results in context of Rough Sets
Chapter 5: Applications
1.1. Results with complete soft metric spaces
1.2 Results with Soft metric spaces with complete soft cone
1.3 Results on Soft Metric space for F-contractive Type Mappings
1.4 Fixed-Point Theorems for Integral Type Mappings Through Rational Expression
3.1 Fixed point Results on a Complete Soft Usual Metric Space
3.2 Common Fixed-Point Theorems Using Rational Contraction in Soft Compact Metric Spaces
3.3. Unique Fixed-Point Theorems related to Soft Banach spaces
4.1. Fixed Point Theorems in Rough Metric Spaces
4.2 Fixed Point Theorems in Compact Rough Metric Spaces
4.3 Fixed Point Theorems in Rough Banach Spaces
5.1. Digital soft α – ψ –contractive type mapping in digital soft metric spaces
5.2. Future scope of (5.1)
5.3 Advanced Manufacturing Process for Automation in Examination System using Artificial Intelligence with Soft Fixed-Point Property.
5.4 Fuzzy Soft Set & Ranking
5.5 An Image Processing System Using Fixed Point and Fuzzy Logic Unit

**"Some Fixed-Point Theorems in The Context of Soft Sets"** is the title of part first, which is subdivided into three chapters. The contents of this part are as follows.

**Chapter1: Fixed Point results Related to Soft Metric spaces**

1.1.: Results with complete soft metric spaces

1.2 Results with Soft metric spaces with complete soft cone

1.3 Results on Soft Metric space for F-contractive Type Mappings

1.4 Fixed-Point Theorems for Integral Type Mappings Through Rational Expression

Chapter -2: Results related to couple soft fixed point

Chapter-3: Results related to Complete soft Usual Metric space, Soft Compact Metric spaces and Soft Banach spaces

3.1 Fixed point Results on a Complete Soft Usual Metric Space

3.2 Common Fixed-Point Theorems Using Rational Contraction in Soft Compact Metric Spaces

3.3. Unique Fixed-Point Theorems on soft $(\psi - \varphi)$ −contractive type mappings in Soft Banach spaces

**Established results are based on the concepts of soft sets and fixed point. Main results of this part are as follows**

**Theo 1.1.1:** Let $( \widetilde{\hat{W}}, \eth_{S_m}, \hat{E} )$ be a $\text{soft}_{\text{complete}_{\text{metric space}}}$. Suppose the $\text{soft}_{\text{mapping}}$ $(\mathrm{F}, \S)$: $( \widetilde{\hat{W}}, \eth_{S_m}, \hat{E} ) \longrightarrow ( \widetilde{\hat{W}}, \eth_{S_m}, \hat{E} )$ satisfies :

$$\widetilde{\eth_{S_m}}\big( (\mathrm{F}, \S)\tilde{a}_{\hat{e}}, (\mathrm{F}, \S)\widetilde{b_{\hat{e},}} \big) \lesssim \tilde{\gamma} \left[\widetilde{\eth_{S_m}}( \tilde{a_{\hat{e}}}, (\mathrm{F}, \S)\tilde{a}_{\hat{e}}) + \widetilde{\eth_{S_m}}( \widetilde{b_{\hat{e},}}, (\mathrm{F}, \S)\widetilde{b_{\hat{e},}}) \right]$$

$$+ \tilde{\zeta} \left[\widetilde{\eth_{S_m}}( \tilde{a_{\hat{e}}}, (\mathrm{F}, \S)\, \widetilde{b_{\hat{e},}}) + \widetilde{\eth_{S_m}}( \widetilde{b_{\hat{e},}}, (\mathrm{F}, \S)( \tilde{a_{\hat{e}}}) \right]$$

$$+ \tilde{\rho} \frac{\left[\widetilde{\eth_{S_m}}( \tilde{a_{\hat{e}}}, (\mathrm{F}, \S)\tilde{a}_{\hat{e}})\, \widetilde{\eth_{S_m}}( \widetilde{b_{\hat{e},}}, (\mathrm{F}, \S)\widetilde{b_{\hat{e},}}) \right]}{\widetilde{\eth_{S_m}}(\tilde{a}_{\hat{e}}, \widetilde{b_{\hat{e},}})}$$

$$+ \widetilde{\omega}\, \widetilde{\eth_{S_m}}(\tilde{a}_{\hat{e}}, \widetilde{b_{\hat{e},}})$$

$\forall\ \tilde{a}_{\hat{e}}, \widetilde{b_{\hat{e},}} \in \widetilde{\hat{W}}$ with $\tilde{a}_{\hat{e}} \neq \widetilde{b_{\hat{e},}}$ and $\widetilde{2\gamma} + \widetilde{2\zeta} + \widetilde{\omega} + \tilde{\rho} \ \widetilde{<}\ \bar{1}$ and $\tilde{\gamma}, \tilde{\zeta}, \tilde{\rho}, \widetilde{\omega}$ all are in $[\bar{0}, \bar{1})$ then $(\mathrm{F}, \S)$ has unique $\mathbb{S}oft_{\mathbb{F}ixed\ \mathbb{P}oint}$.

**Theorem 1.1.2:** Let $( \widetilde{\hat{W}}, \eth_{S_m}, \hat{E} )$ be a $\mathbb{S}oft\ \ \mathbb{M}etric_{spaces}$. The $\text{soft}_{\text{mapping}}$

$(\mathbb{T}, \S)$: $(\widetilde{\mathbb{W}}, \eth_{S_m}, \hat{\mathbb{E}}) \longrightarrow (\widetilde{\mathbb{W}}, \eth_{S_m}, \hat{\mathbb{E}})$ is given. For some $p \in \mathbb{N}$, $(\mathbb{T}, \S)^{\tilde{P}}$ is cont. then $(\mathbb{T}, \S)$ will have unique $\mathbb{S}oft_{\mathbb{F}ixed\ \mathbb{P}oint}$.

***Theorem 1.1.4:*** *Let* $(\widetilde{\mathbb{W}}, \eth_{S_m}, \hat{\mathbb{E}})$ *be a* $soft_{complete_{metric\ space}}$. Suppose the $\text{soft}_{\text{self m}_{\text{appings}}}$ $(\mathbb{A}, \S^*)$, $(\mathbb{B}, \S)$, $(\mathbb{D}, \S)$, $(\mathbb{M}, \S)$, $(\mathbb{\Phi}, \S)$, $(\mathbb{T}, \S)$ defined

on$(\widetilde{\mathbb{W}}, \eth_{S_m}, \hat{\mathbb{E}})$ as :

*[1.1.4 *]: $(\mathbb{\Phi}, \S)\tilde{a}_{\hat{e}} \subseteq (\mathbb{B}, \S)(\mathbb{D}, \S)\tilde{a}_{\hat{e}}$, $\quad (\mathbb{T}, \S)\tilde{a}_{\hat{e}} \subseteq (\mathbb{A}, \S^*)(\mathbb{M}, \S)\tilde{a}_{\hat{e}}$*

*[1.1.4 **]:* $\forall\ \tilde{a}_{\hat{e}}, \widetilde{b_{\hat{e}}} \in \widetilde{\mathbb{W}}$, $\tilde{a}_{\hat{e}} \neq \widetilde{b_{\hat{e}}}$, $\tilde{\kappa}$ , $\tilde{\zeta}$, $\tilde{\gamma}$, $\tilde{\rho}$, $\tilde{\omega} \in \widetilde{I^+}$, *are taken as*

$\tilde{\kappa} + \tilde{\zeta} + 2\tilde{\gamma} + \tilde{\rho} + \tilde{\lesssim} \tilde{1}$ *for which*

$$\widetilde{\eth_{S_m}}\left((\mathbb{T}, \S)\widetilde{b_{\hat{e}}}, (\mathbb{A}, \S^*)(\mathbb{M}, \S)\widetilde{a_{\hat{e}}}\right) + \widetilde{\eth_{S_m}}\left((\mathbb{\Phi}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{B}, \S)(\mathbb{D}, \S)\widetilde{b_{\hat{e}}}\right)$$

$$+ \widetilde{\eth_{S_m}}\left((\mathbb{B}, \S)(\mathbb{D}, \S)b_{\hat{e}}, \widetilde{(\mathbb{A}, \S)(\mathbb{M}, \S)}\widetilde{a_{\hat{e}}}\right) \neq \tilde{0},$$

*or* $\widetilde{\eth_{S_m}}\left((\mathbb{\Phi}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{T}, \S)\widetilde{b_{\hat{e}}}\right) = \tilde{0}$, *if*

$$\widetilde{\eth_{S_m}}\left((\mathbb{T}, \S)\widetilde{b_{\hat{e}}}, (\mathbb{A}, \S^*)(\mathbb{M}, \S)\widetilde{a_{\hat{e}}}\right) + \widetilde{\eth_{S_m}}\left((\mathbb{\Phi}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{B}, \S)(\mathbb{D}, \S)\widetilde{b_{\hat{e}}}\right)$$

$$+ \widetilde{\eth_{S_m}}\left((\mathbb{B}, \S)(\mathbb{D}, \S)b_{\hat{e}}, \widetilde{(\mathbb{A}, \S)(\mathbb{M}, \S)}\widetilde{a_{\hat{e}}}\right) = \tilde{0},$$ *Such that*

$$\widetilde{\eth_{S_m}}\left((\mathbb{\Phi}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{T}, \S)\widetilde{b_{\hat{e}}}\right) \tilde{\leq} \tilde{\kappa}\ \widetilde{\eth_{S_m}}\left((\mathbb{A}, \S^*)(\mathbb{M}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{B}, \S)(\mathbb{D}, \S)\widetilde{b_{\hat{e}}}\right)$$

$$+\tilde{\zeta}\ max\left\{\begin{matrix}\frac{\widetilde{\eth_{S_m}}\left((\mathbb{A}, \S^*)(\mathbb{M}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{B}, \S)(\mathbb{D}, \S)\widetilde{b_{\hat{e}}}\right)}{1+\widetilde{\eth_{S_m}}\left((\mathbb{A}, \S^*)(\mathbb{M}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{B}, \S)(\mathbb{D}, \S)\widetilde{b_{\hat{e}}}\right)}, \\ \frac{\widetilde{\eth_{S_m}}\left((\mathbb{A}, \S^*)(\mathbb{M}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{\Phi}, \S)\widetilde{a_{\hat{e}}}\right)}{1+\widetilde{\eth_{S_m}}\left((\mathbb{A}, \S^*)(\mathbb{M}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{\Phi}, \S)\widetilde{a_{\hat{e}}}\right)}, \\ \frac{\widetilde{\eth_{S_m}}\left((\mathbb{\Phi}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{B}, \S)(\mathbb{D}, \S)\widetilde{b_{\hat{e}}}\right)}{1+\widetilde{\eth_{S_m}}\left((\mathbb{\Phi}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{B}, \S)(\mathbb{D}, \S)\widetilde{b_{\hat{e}}}\right)}\end{matrix}\right\}+$$

$$\tilde{\gamma}\ \left\{\frac{\widetilde{\eth_{S_m}}\left((\mathbb{B}, \S)(\mathbb{D}, \S)b_{\hat{e}}, \widetilde{(\mathbb{T}, \S)}\widetilde{b_{\hat{e}}}\right) + \widetilde{\eth_{S_m}}\left((\mathbb{T}, \S)\widetilde{b_{\hat{e}}}, (\mathbb{A}, \S^*)(\mathbb{M}, \S)\widetilde{a_{\hat{e}}}\right) + \widetilde{\eth_{S_m}}\left((\mathbb{\Phi}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{B}, \S)(\mathbb{D}, \S)\widetilde{b_{\hat{e}}}\right)}{1 + \widetilde{\eth_{S_m}}\left((\mathbb{B}, \S)(\mathbb{D}, \S)b_{\hat{e}}, \widetilde{(\mathbb{T}, \S)}\widetilde{b_{\hat{e}}}\right)\widetilde{\eth_{S_m}}\left((\mathbb{T}, \S)\widetilde{b_{\hat{e}}}, (\mathbb{A}, \S^*)(\mathbb{M}, \S)\widetilde{a_{\hat{e}}}\right)\widetilde{\eth_{S_m}}\left((\mathbb{\Phi}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{B}, \S)(\mathbb{D}, \S)\widetilde{b_{\hat{e}}}\right)}\right\}$$

$$+\tilde{\rho}\frac{\widetilde{\eth_{S_m}}\left((\mathbb{\Phi}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{B}, \S)(\mathbb{D}, \S)\widetilde{b_{\hat{e}}}\right)+\widetilde{\eth_{S_m}}\left((\mathbb{B}, \S)(\mathbb{D}, \S)\widetilde{b_{\hat{e}}}, (\mathbb{A}, \S^*)(\mathbb{M}, \S)\widetilde{a_{\hat{e}}}\right)}{1+\widetilde{\eth_{S_m}}\left((\mathbb{\Phi}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{B}, \S)(\mathbb{D}, \S)\widetilde{b_{\hat{e}}}\right)\widetilde{\eth_{S_m}}\left((\mathbb{B}, \S)(\mathbb{D}, \S)b_{\hat{e}}, \widetilde{(\mathbb{T}, \S)}\widetilde{b_{\hat{e}}}\right)\widetilde{\eth_{S_m}}\left((\mathbb{A}, \S^*)(\mathbb{M}, \S)\widetilde{a_{\hat{e}}}, (\mathbb{\Phi}, \S)\widetilde{a_{\hat{e}}}\right)} \qquad (1.1.4(i))$$

*The pair* $((\mathbb{A})(\mathbb{M}), \mathbb{\Phi})$ *and* $((\mathbb{B},)(\mathbb{D}), \mathbb{T})$ *are commute as well as compatible then*

*(𝔄), (𝔅), (𝔇) ,(𝔐) , (𝔉) have a unique* $_{common\ soft\ fixed\ point}$

***Theorem 1.1.5****: Let* $(\widetilde{\hat{W}}, \eth_{S_m}, \hat{E})$ *be a* complex valuied $_{soft\ metric\ space}$. *Suppose the* $soft_{selfm_{appingS}}$ $(\mathfrak{A}, \S^*)$, $(\mathfrak{B}, \S)$, $(\mathfrak{D}, \S)$, $(\mathfrak{M}, \S)$, $(\mathfrak{P}, \S)$, $(\mathfrak{T}, \S)$ *defined on*$(\widetilde{\hat{W}}, \eth_{S_m}, \hat{E})$ *as* :

[1.1.5 *]: $(\mathfrak{P}, \S)\tilde{a}_{\hat{e}} \subset (\mathfrak{B}, \S)(\mathfrak{D}, \S)\tilde{a}_{\hat{e}}$ , $(\mathfrak{T}, \S)\tilde{a}_{\hat{e}} \subset (\mathfrak{A}, \S^*)(\mathfrak{M}, \S)\tilde{a}_{\hat{e}}$

[1.1.5 **]:$\forall\ \tilde{a}_{\hat{e}}, \widetilde{b_{\hat{e}'}} \in \widetilde{\hat{W}}, \tilde{a}_{\hat{e}} \neq \widetilde{b_{\hat{e}'}}$, $\tilde{\kappa}$ , $\tilde{\zeta}$ , $\tilde{y}$, $\tilde{\rho}$, $\tilde{\omega} \in \tilde{I}^{+}$, are taken as

$\tilde{\kappa} + \tilde{\zeta} + 2\tilde{y} + \tilde{\rho} + \lesssim \tilde{1}$ for which

$$\eth_{\widetilde{S_m}}\left((\mathfrak{T}, \S)\widetilde{b_{\hat{e}'}}, (\mathfrak{A}, \S^*)(\mathfrak{M}, \S)\tilde{a}_{\hat{e}}\right) + \eth_{\widetilde{S_m}}\left((\mathfrak{P}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{B}, \S)(\mathfrak{D}, \S)\widetilde{b_{\hat{e}'}}\right)$$

$$+ \eth_{\widetilde{S_m}}\left((\mathfrak{B}, \S)(\mathfrak{D}, \S)b_{\hat{e}'}, \widetilde{(\mathfrak{A}, \S)(\mathfrak{M}, \S)}\tilde{a}_{\hat{e}}\right) \neq \tilde{0},$$

or $\eth_{\widetilde{S_m}}\left((\mathfrak{P}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{T}, \S)\widetilde{b_{\hat{e}'}}\right) = \tilde{0}$, if

$$\eth_{\widetilde{S_m}}\left((\mathfrak{T}, \S)\widetilde{b_{\hat{e}'}}, (\mathfrak{A}, \S^*)(\mathfrak{M}, \S)\tilde{a}_{\hat{e}}\right) + \eth_{\widetilde{S_m}}\left((\mathfrak{P}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{B}, \S)(\mathfrak{D}, \S)\widetilde{b_{\hat{e}'}}\right)$$

$$+ \eth_{\widetilde{S_m}}\left((\mathfrak{B}, \S)(\mathfrak{D}, \S)b_{\hat{e}'}, \widetilde{(\mathfrak{A}, \S)(\mathfrak{M}, \S)}\tilde{a}_{\hat{e}}\right) = \tilde{0},$$ Such that

$$\eth_{\widetilde{S_m}}\left((\mathfrak{P}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{T}, \S)\widetilde{b_{\hat{e}'}}\right) \lesssim \tilde{\kappa}\ \eth_{\widetilde{S_m}}\left((\mathfrak{A}, \S^*)(\mathfrak{M}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{B}, \S)(\mathfrak{D}, \S)\widetilde{b_{\hat{e}'}}\right)$$

$$+\tilde{\zeta}\ \max\left\{\frac{\eth_{\widetilde{S_m}}\left((\mathfrak{A}, \S^*)(\mathfrak{M}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{B}, \S)(\mathfrak{D}, \S)\widetilde{b_{\hat{e}'}}\right)}{1+\eth_{\widetilde{S_m}}\left((\mathfrak{A}, \S^*)(\mathfrak{M}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{B}, \S)(\mathfrak{D}, \S)\widetilde{b_{\hat{e}'}}\right)}, \frac{\eth_{\widetilde{S_m}}\left((\mathfrak{A}, \S)(\mathfrak{M}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{P}, \S)\tilde{a}_{\hat{e}}\right)}{1+\eth_{\widetilde{S_m}}\left((\mathfrak{A}, \S)(\mathfrak{M}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{P}, \S)\tilde{a}_{\hat{e}}\right)}, \frac{\eth_{\widetilde{S_m}}\left((\mathfrak{P}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{B}, \S)(\mathfrak{D}, \S)\widetilde{b_{\hat{e}'}}\right)}{1+\eth_{\widetilde{S_m}}\left((\mathfrak{P}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{B}, \S)(\mathfrak{D}, \S)\widetilde{b_{\hat{e}'}}\right)}\right\}+$$

$$\tilde{y}\ \left\{\frac{\eth_{\widetilde{S_m}}\left((\mathfrak{B}, \S)(\mathfrak{D}, \S)b_{\hat{e}}, \widetilde{(\mathfrak{T}, \S)}\widetilde{b_{\hat{e}'}}\right) + \eth_{\widetilde{S_m}}\left((\mathfrak{T}, \S)\widetilde{b_{\hat{e}'}}, (\mathfrak{A}, \S)(\mathfrak{M}, \S)\tilde{a}_{\hat{e}}\right) + \eth_{\widetilde{S_m}}\left((\mathfrak{P}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{B}, \S)(\mathfrak{D}, \S)\widetilde{b_{\hat{e}'}}\right)}{1 + \eth_{\widetilde{S_m}}\left((\mathfrak{B}, \S)(\mathfrak{D}, \S)b_{\hat{e}}, \widetilde{(\mathfrak{T}, \S)}\widetilde{b_{\hat{e}'}}\right)\eth_{\widetilde{S_m}}\left((\mathfrak{T}, \S)\widetilde{b_{\hat{e}'}}, (\mathfrak{A}, \S)(\mathfrak{M}, \S)\tilde{a}_{\hat{e}}\right)\eth_{\widetilde{S_m}}\left((\mathfrak{P}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{B}, \S)(\mathfrak{D}, \S)\widetilde{b_{\hat{e}'}}\right)}\right\}$$

$$+\tilde{\rho}\ \frac{\eth_{\widetilde{S_m}}\left((\mathfrak{P}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{B}, \S)(\mathfrak{D}, \S)\widetilde{b_{\hat{e}'}}\right)+\eth_{\widetilde{S_m}}\left((\mathfrak{B}, \S)(\mathfrak{D}, \S)\widetilde{b_{\hat{e}'}}, (\mathfrak{A}, \S)(\mathfrak{M}, \S)\tilde{a}_{\hat{e}}\right)}{1+\eth_{\widetilde{S_m}}\left((\mathfrak{P}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{B}, \S)(\mathfrak{D}, \S)\widetilde{b_{\hat{e}'}}\right)\eth_{\widetilde{S_m}}\left((\mathfrak{B}, \S)(\mathfrak{D}, \S)b_{\hat{e}}, \widetilde{(\mathfrak{T}, \S)}\widetilde{b_{\hat{e}'}}\right)\eth_{\widetilde{S_m}}\left((\mathfrak{A}, \S)(\mathfrak{M}, \S)\tilde{a}_{\hat{e}}, (\mathfrak{P}, \S)\tilde{a}_{\hat{e}}\right)} \quad (1.15.\ (i))$$

(1.1.5***) $(\mathfrak{A}, \S^*)(\mathfrak{M}, \S)$ are compatible and $(\mathfrak{A}, \S^*)(\mathfrak{M}, \S)$ or $(\mathfrak{P}, \S$ is continuous and

, $(\mathfrak{B}, \S)(\mathfrak{D}, \S)$, $\mathfrak{T}$ ) are weakly compatible

(1.1.5****)($\mathfrak{B}$ $\mathfrak{D}$, $\mathfrak{T}$) are compatible and $\mathfrak{B}$ $\mathfrak{D}$ or $\mathfrak{T}$ is continuous and or $(\mathfrak{A}(\mathfrak{M})\mathfrak{P}, )$are weakly compatible , then $(\mathfrak{A})$, $(\mathfrak{B})$, $(\mathfrak{D})$ ,$(\mathfrak{M})$ , $(\mathfrak{P})$ have a unique $_{common\ soft\ fixed\ point}$.

## 1.2 Results with $\mathbb{S}\mathbf{oft}_{\mathbf{metric}}$ spaces with complete $soft_{cone}$

In this part the existence and uniqueness of a $\mathbb{F}ixed_{\mathbb{P}oint}$ in a $\mathbb{S}oft_{Cone}\,\mathbb{M}etric_{space}$ are discussed for $\mathbb{S}elf_{mapping}$ using expanding and comparison function in the setting of $Cone\_\mathbb{M}etric_{space}$ for $randam_{\,operator}$. About $Cone\_\mathbb{M}etric_{space}$ one can refer [54,105]. $\mathbb{S}oft_{Set\ Theory}$ $\mathbb{S}oft_{\mathbb{M}etric_{space}}$ *already have been discussed.* $\mathbb{S}oft_{Cone}\,\mathbb{M}etric_{space}$, may be seen in [1-3,5-12, 14]. The details on $\mathbb{F}ixed_{\mathbb{P}oint_{Theorems}}$ for $randam_{\,operator}$ can be viewed in [26,27,34]. The definition of $convergence_{\,\mathbb{S}equence}$ $\mathbb{C}auchy_{\mathbb{S}equence}$, completeness and $\mathbb{S}oft_{\mathbb{B}anach_{\,\mathbb{S}pace}}$ can be modified for $randam_{\,operator}$ easily. The established results are motivated by [58]

**Theorem 1.2. 1:** Assume that $(\widetilde{\hat{W}}, \widetilde{\eth_{S_m}})$ is $\mathbb{C}omplete_{\,\mathbb{S}oft_{\,\mathbb{M}etric\,space}}$ with set of parameters $\widetilde{\hat{E}}$ .Soft cone (Q,$\mathbb{A}$) is regular $\xi$ is a measerable selector and $(\Omega, \Sigma)$ denotes a measurable space consisting of a set $\Omega$ and sigma algebra $\Sigma$ of subset of $\Omega$. $\widetilde{\mathfrak{F}}$ = $(\mathfrak{T}, \S)$,

$\widetilde{\mathfrak{F}}$ : $\widetilde{\hat{W}} \to \widetilde{\hat{W}}$ satisfies the condition

$$\widetilde{\eth_{S_m}}\left(\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathfrak{U}_e}\,,\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathcal{V}_e}\right) \le \tilde{\zeta}\,\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathfrak{U}_e}\,,\widetilde{\xi\mathcal{V}_e}\right)$$

$$+\tilde{\gamma}\left[\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathfrak{U}_e}\,,\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathcal{V}_e}\right)+\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathcal{V}_e},\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathfrak{U}_e}\right)\right]$$

$$+\ \tilde{\omega},\ \frac{\widetilde{\eth_{S_m}}(\widetilde{\xi\mathfrak{U}_e}\,\widetilde{\xi},\widetilde{\mathcal{V}_e})+\left[\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathfrak{U}_e},\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathcal{V}_e}\right)+\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathcal{V}_e},\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathfrak{U}_e}\right)\right]}{1+\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathfrak{U}_e},\widetilde{\xi\mathcal{V}_e}\right)\left[\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathfrak{U}_e},\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathcal{V}_e}\right)\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathcal{V}_e},\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathfrak{U}_e}\right)\right]}$$

$$+\ \tilde{\delta}\left[\frac{\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathfrak{U}_e},\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathfrak{U}_e}\right)+\widetilde{\eth_{S_m}}\left(\widetilde{\xi y},\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathcal{V}_e}\right)+\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathfrak{U}_e},\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathcal{V}_e}\right)+\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathcal{V}_e},\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathfrak{U}_e}\right)}{1+\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathfrak{U}_e},\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathfrak{U}_e}\right)\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathcal{V}_e},\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathcal{V}_e}\right)\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathfrak{U}_e},\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathcal{V}_e}\right)\widetilde{\eth_{S_m}}\left(\widetilde{\xi\mathcal{V}_e},\widetilde{\mathfrak{F}}\,\widetilde{\xi\mathfrak{U}_e}\right)}\right]$$

[1.2.1(i) ]

$\forall\quad \widetilde{\xi\mathfrak{U}_e},\widetilde{\xi\mathcal{V}_e} \in \widetilde{\hat{W}}$ and $\tilde{\zeta}\ ,\tilde{\gamma},\tilde{\delta},\widetilde{\tilde{\omega}} \ge \tilde{0}\ Such_{That}$

$\tilde{0} \le (\tilde{\gamma} + 2\widetilde{\delta + \tilde{\omega}}) + \frac{\tilde{\zeta}+\tilde{\omega}}{\tilde{2}} < \frac{1}{2}$. Then $\widetilde{\mathfrak{F}}$ = $(\mathfrak{T}, \S)$ has unique $\mathbb{S}oft\_random_{\,\mathbb{F}ixed\ \mathbb{P}oint}$ in $\widetilde{\hat{W}}$.

**Theorem 1.2.2:** Assume that $(\widetilde{\hat{W}}, \widetilde{\eth_{S_m}})$ is $\mathbb{C}omplete_{\,\mathbb{S}oft_{\,\mathbb{M}etric\,space}}$ with set of parameters $\widetilde{\hat{E}}$ .Soft cone (Q,$\mathbb{A}$) is regular $\xi$ is a measerable selector and $(\Omega, \Sigma)$ denotes a measurable space consisting of a set $\Omega$ and sigma algebra $\Sigma$ of subset of $\Omega$. If $\widetilde{\mathfrak{F}}$ , $\widetilde{\mathfrak{G}}$: $\widetilde{\hat{W}} \to \widetilde{\hat{W}}$ , where $\widetilde{\mathfrak{F}} = (\mathrm{T}, \S)$ , $\widetilde{\mathfrak{G}} = (\mathcal{R}, \S)$ and satisfies the condition,

$$\widetilde{\eth_{S_m}}\left(\widetilde{\mathfrak{F}}\,\tilde{\xi}\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{G}}\tilde{\xi}\widetilde{\mathcal{V}_e}\right) \cong \tilde{\zeta}\,\widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathfrak{U}_e}, \widetilde{\mathcal{V}_e}\right)$$

$$+ \tilde{\mathcal{Y}}\left[\widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{G}}\tilde{\xi}\widetilde{\mathcal{V}_e}\right) + \widetilde{\eth_{S_m}}\left(\tilde{\xi}\tilde{y}, \widetilde{\mathfrak{F}}\ \tilde{\xi}\widetilde{\mathfrak{U}_e}\right)\right]$$

$$+\tilde{\omega}\frac{\widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathfrak{U}_e}, \tilde{\xi}\widetilde{\mathcal{V}_e}\right) + \left[\widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{G}}\tilde{\xi}\widetilde{\mathcal{V}_e}\right) + \widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathcal{V}_e}, \widetilde{\mathfrak{F}}\ \tilde{\xi}\widetilde{\mathfrak{U}_e}\right)\right]}{1+\widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathfrak{U}_e}, \tilde{\xi}\widetilde{\mathcal{V}_e}\right)\left[\widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{G}}\tilde{\xi}\widetilde{\mathcal{V}_e}\right)\widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathcal{V}_e}, \widetilde{\mathfrak{F}}\ \tilde{\xi}\widetilde{\mathfrak{U}_e}\right)\right]}$$

$$+\tilde{\delta}\left[\frac{\widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{F}}\ \tilde{\xi}\widetilde{\mathfrak{U}_e}\right) + \widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathcal{V}_e}, \widetilde{\mathfrak{G}}\tilde{\xi}\widetilde{\mathcal{V}_e}\right) + \widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{G}}\tilde{\xi}\widetilde{\mathcal{V}_e}\right) + \widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathcal{V}_e}, \widetilde{\mathfrak{F}}\ \tilde{\xi}\mathfrak{u}_e\right)}{1+\widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{F}}\ \tilde{\xi}\widetilde{\mathfrak{U}_e}\right)\widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathcal{V}_e}, \widetilde{\mathfrak{G}}\tilde{\xi}\widetilde{\mathcal{V}_e}\right)\widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{G}}\tilde{\xi}\widetilde{\mathcal{V}_e}\right)\widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathcal{V}_e}, \widetilde{\mathfrak{F}}\ \tilde{\xi}\widetilde{\mathfrak{U}_e}\right)}\right] \qquad [1.2.2(i)]$$

For every $\tilde{\xi}\widetilde{\mathfrak{U}_e}, \tilde{\xi}\widetilde{\mathcal{V}_e} \cong \widetilde{\hat{W}}$ and $\tilde{\zeta}$ , $\tilde{\mathcal{Y}}, \tilde{\delta}, \tilde{\omega} \cong \tilde{0}$ $Such_{That}$

$\tilde{0} \cong (\tilde{\mathcal{Y}} + 2\widetilde{\delta + \omega}) + \frac{\tilde{\zeta}+\tilde{\omega}}{\tilde{2}} < \frac{\tilde{1}}{\tilde{2}}$. Then $\widetilde{\mathfrak{F}}$ , $\widetilde{\mathfrak{G}}$ has common $\mathbb{S}oft\_random_{\ \mathbb{F}ixed\ \mathbb{P}oint}$ in $\widetilde{\hat{W}}$.

**Theorem 1.2.3:** Assume that $(\widetilde{\hat{W}}, \widetilde{\eth_{S_m}})$ is $\mathbb{C}omplete_{\,\mathbb{S}oft_{\,\mathbb{M}etric\,space}}$ with set of parameters $\widetilde{\hat{E}}$ . $\mathbb{S}oft_{Cone}$ (Q,$\mathbb{A}$) is regular $\xi$ is a $\mathrm{measerable}_{\mathrm{selector}}$

and $(\Omega, \Sigma)$ denotes a $\mathrm{measerable}_{\mathrm{space}}$ consisting of a set $\Omega$ and sigma algebra $\Sigma$ of subset of $\Omega$ . $\widetilde{\mathfrak{F}}$ , $\widetilde{\mathfrak{G}}$ and $\widetilde{\mathfrak{H}}$from $\widetilde{\hat{W}}$ into itself

where $\widetilde{\mathfrak{F}} = (\mathrm{T}, \S)$ , $\widetilde{\mathfrak{G}} = (\mathcal{R}, \S)$ $\widetilde{\mathfrak{H}} = (\mathcal{S}, \S)$ satisfies the condition

$$\widetilde{\eth_{S_m}}\left(\widetilde{\mathfrak{F}}\ \widetilde{\mathfrak{G}}\xi\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{H}}\widetilde{\mathfrak{G}}\xi\widetilde{\mathcal{V}_e}\right) \cong \tilde{\zeta}\,\widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathfrak{U}_e}, \xi\widetilde{\mathcal{V}_e}\right)$$

$$+ \tilde{\mathcal{Y}}\left[\widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{G}}\tilde{\xi}\mathcal{V}_e\right) + \widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathcal{V}_e}, \widetilde{\mathfrak{F}}\ \widetilde{\mathfrak{G}}\xi\widetilde{\mathfrak{U}_e}\right)\right]$$

$$+\tilde{\omega}\frac{\widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathfrak{U}_e}, \xi\widetilde{\mathcal{V}_e}\right) + \left[\widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{G}}\xi\widetilde{\mathcal{V}_e}\right) + \widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathcal{V}_e}, \widetilde{\mathfrak{F}}\ \widetilde{\mathfrak{G}}\xi\widetilde{\mathfrak{U}_e}\right)\right]}{1+\widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathfrak{U}_e}, \xi\widetilde{\mathcal{V}_e}\right) + \left[\widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{H}}\widetilde{\mathfrak{G}}\xi\widetilde{\mathcal{V}_e}\right) + \widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathcal{V}_e}, \widetilde{\mathfrak{F}}\ \widetilde{\mathfrak{G}}\xi\widetilde{\mathfrak{U}_e}\right)\right]}$$

$$+\tilde{\delta}\left[\frac{\widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{F}}\ \widetilde{\mathfrak{G}}\xi\widetilde{\mathfrak{U}_e}\right) + \widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathcal{V}_e}, C\widetilde{\mathfrak{G}}\xi\widetilde{\mathcal{V}_e}\right) + \widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{H}}\widetilde{\mathfrak{G}}\xi\widetilde{\mathcal{V}_e}\right) + \widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathcal{V}_e}, \widetilde{\mathfrak{F}}\ \widetilde{\mathfrak{G}}\xi\widetilde{\mathfrak{U}_e}\right)}{1+\widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{F}}\ \widetilde{\mathfrak{G}}\xi\widetilde{\mathfrak{U}_e}\right)\widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathcal{V}_e}, \widetilde{\mathfrak{H}}\widetilde{\mathfrak{G}}\xi\widetilde{\mathcal{V}_e}\right)\widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathfrak{U}_e}, \widetilde{\mathfrak{H}}\widetilde{\mathfrak{G}}\xi\widetilde{\mathcal{V}_e}\right)\widetilde{\eth_{S_m}}\left(\xi\widetilde{\mathcal{V}_e}, \widetilde{\mathfrak{F}}\ \widetilde{\mathfrak{G}}\xi\widetilde{\mathfrak{U}_e}\right)}\right] \quad [1.2.3.(i)]$$

$\forall\, \tilde{\xi}\widetilde{\mathcal{U}}_e, \tilde{\xi}\widetilde{\mathcal{V}}_e \Subset \widetilde{\hat{W}}$ and $\tilde{\varsigma}\ , \tilde{\gamma}, \tilde{\delta}, \widetilde{\omega} \cong \tilde{0}\ Such_{That}, \tilde{0} \lessapprox (\tilde{\gamma} + 2\widetilde{\delta + \omega}) + \frac{\tilde{\varsigma}+\widetilde{\omega}}{\tilde{2}} \lessapprox \frac{1}{\tilde{2}}.$ ,

$\Rightarrow \widetilde{\mathfrak{F}}$ , $\widetilde{\mathfrak{G}}$ and $\widetilde{\mathfrak{H}}$ has $\mathbb{U}nique_{\mathbb{S}oft_{\mathbb{R}andom}}\ \mathbb{F}ixed_{\mathbb{P}oint}$ in $\widetilde{\hat{W}}$

## 1.3 RESULTS ON SOFT METRIC SPACE FOR F-CONTRACTIVE TYPE MAPPINGS

The present part deals with $\mathbb{F}ixed_{\ Point}$ results in $\mathbb{S}oft_{\ metric_{space}}$ for $F_{contractive}$ type functions for $random_{\ operator}$. Wardowski [110-111] introduced $F_{contraction}$ and established $\mathbb{F}ixed_{\ Point}$ theorems in this regard. The detail about $F_{contraction}$ may viewed in[6-8] and regarding $Suzuki_{\ Type}$ representation may refer in [107-108].More on $\mathbb{F}ixed_{\ Point}$ theory for $Random_{\ Operator}$ and convergence can be studied in [25-27,34]. For this part $(\Omega, \Sigma)$ represents a measurable $_{space}$ which consists a set $\Omega$ , sigma_ algebra $\Sigma \sqsubset \Omega$ .

**Theorem 1.3.1 :** Let $\left(\widetilde{\hat{W}}, \widetilde{\eth_{S_m}}, E\right)$ be a $\mathbb{C}omplete_{\mathbb{S}oft}\ \mathbb{M}etric_{space}$ and let $(\mathcal{P}, \varphi)$ be a $\mathbb{S}elf_{mapping}$ of a $\widetilde{\hat{W}}$ into itself. $\xi$ is a measerable$_{\text{selector}}$ and $(\Omega, \Sigma)$ denotes a measerable$_{\text{space}}$ consisting of a set $\Omega$ and sigma algebra $\Sigma \Subset \Omega$ . Suppose $F \Subset \mathcal{F}$ and there exists $\tau \gtrsim 0$ such that for all $\tilde{\xi}\widetilde{\mathcal{U}}_\lambda, \tilde{\xi}\tilde{\mathcal{V}}_\mu \Subset (\widetilde{\hat{W}})$

$$\widetilde{\eth_{S_m}}\left((\mathcal{P}, \varphi)\left(\tilde{\xi}\widetilde{\mathcal{U}}_\lambda\right), (\mathcal{P}, \varphi)\left(\tilde{\xi}\widetilde{\mathcal{V}}_\mu\right)\right) \gtrsim 0$$

$$\Rightarrow \tau + F\left(\widetilde{\eth_{S_m}}\left((\mathcal{P}, \varphi)\left(\tilde{\xi}\widetilde{\mathcal{U}}_\lambda\right), (\mathcal{P}, \varphi)\left(\tilde{\xi}\widetilde{\mathcal{V}}_\mu\right)\right)\right) \cong F\left(\widetilde{\eth_{S_m}}\left(\tilde{\xi}\widetilde{\mathcal{U}}_\lambda, \tilde{\xi}\widetilde{\mathcal{V}}_\mu\right)\right)$$

(1.3.1.(i) )

Then $(\mathcal{P}, \varphi)$ has a unique $random_{\ \mathbb{S}oft}\ Fixed_{\ point}$ in $\widetilde{\hat{W}}$ and for every $\tilde{\xi}\ \tilde{\mathcal{U}}_\lambda^* \Subset \widetilde{\hat{W}}$ the sequence $(\mathcal{P}, \varphi)^n\left(\tilde{\xi}\tilde{\mathcal{U}}_{\lambda_0}^0\right)\}$ converge to $\tilde{\xi}\ \tilde{\mathcal{U}}_\lambda^*$.

**Theorem 1.3.2: :** Let $\left(\widetilde{\hat{W}}, \widetilde{\eth_{S_m}}, E\right)$ be a $\mathbb{C}omplete_{\mathbb{S}oft}\ \mathbb{M}etric_{space}$

and let $(\mathcal{P}, \varphi)$ be a be generalized F-Suzuki contraction of a $\widetilde{\hat{W}}$ into itself. $\xi$ is a $\text{measerable}_{\text{selector}}$ and $(\Omega, \Sigma)$ denotes a $\text{measerable}_{\text{space}}$ consisting of a set $\Omega$ and sigma algebra $\Sigma \sqsubset \Omega$ . Then $(\mathcal{P}, \varphi)$ has a unique random $soft_{fixed\ point}$ in $\widetilde{\hat{W}}$.

Let $\left(\widetilde{\hat{W}}, \widetilde{\delta_{S_m}}, E\right)$ be a $complete_{soft_{metric}}$ space and let $(f, \varphi)$ be generalized F-Suzuki contraction of $\widetilde{\hat{W}}$ into itself. $\xi$ is a measerable selector and $(\Omega, \Sigma)$ denotes a measurable space consisting of a set $\Omega$ and sigma algebra $\Sigma$ of subset of $\Omega$ Then $(\mathcal{P}, \varphi)$ has a unique random $\mathbb{S}oft_{\mathbb{F}ixed\ \mathbb{P}oint}$ in $\widetilde{\hat{W}}$.

## 1.4 FIXED-POINT THEOREMS FOR INTEGRAL TYPE MAPPINGS THROUGH RATIONAL EXPRESSION

M. S. Khan,M. Swalech and S. Sessa [62] introduced the concept of $\mathbb{A}ltering\ _{\mathbb{D}istence_{\mathbb{F}unction}}$ which is a control function that alters distance between two points in a metric space. Branciari [9] established a $\mathbb{F}ixed\ _{\mathbb{P}oint_{theorem}}$ for a self mapping which satisfying the concept of $\mathbb{B}anach\ _{contraction\ _{principl}}$ specially for $integral_{type_{mappings}}$. Taking this concept many authors generalized $\mathbb{B}anach\ _{contraction\ _{principle}}$ for $integral_{type_{inequalities}}$ .for various $\mathbb{C}ontractive\ _{conditions}$ of $integral_{type}$

Work in this part is related to some $\mathbb{S}oft_{\mathbb{F}ixed\ \mathbb{P}oint}$ results for $rational\ _{expressions}$ using $\mathbb{A}ltering\ _{\mathbb{D}istence_{\mathbb{F}unction}}$ for $\mathbb{C}omplete_{\mathbb{S}oft}\ \mathbb{M}etric_{space}$. Obtained results are motivated by Molodtsov[68] , Khan M.S. et.al [62] . and Braninciari [9] . Main results of this chapters are as follows:

**Theorem 1.4.1 :** Let $\left(\widetilde{\hat{W}}, \widetilde{\delta_{S_m}}, \widetilde{\hat{E}}\right)$ be a $\mathbb{S}oft_{Complete\ \mathbb{M}etric_{\mathbb{S}pace}}$. Suppose the $\mathbb{S}oft_{\mathbb{M}apping}$ $(f, \varphi) : \left(\widetilde{\hat{W}}, \widetilde{\delta_{S_m}}, \widetilde{\hat{E}}\right) \to \left(\widetilde{\hat{W}}, \widetilde{\delta_{S_m}}, \widetilde{\hat{E}}\right)$ satisfies the $\mathbb{S}oft_{\mathbb{C}ontractive\ _{Condition}}$

$$\widetilde{\Psi}\left[\widetilde{\delta_{S_m}}\left((f, \varphi)(\widetilde{\mathfrak{M}}_\lambda), (f, \varphi)(\widetilde{\mathfrak{N}}_\mu)\right)\right] \lesssim$$

$$\psi\left[\widetilde{\mathcal{M}}(\widetilde{\mathfrak{M}}_\lambda, \widetilde{\mathfrak{N}}_\mu)\right] - \varphi\left[\widetilde{\mathcal{M}}(\widetilde{\mathfrak{M}}_\lambda, \widetilde{\mathfrak{N}}_\mu)\right] \qquad (1.4.1.(i))$$

$\forall\ \widetilde{\mathfrak{M}}_\lambda, \widetilde{\mathfrak{N}}_\mu \in \widetilde{\hat{W}}, \widetilde{\mathfrak{M}}_\lambda \neq \widetilde{\mathfrak{N}}_\mu$, where $\widetilde{\Psi}, \varphi$ are $\mathbb{A}ltering\ _{\mathbb{D}istence_{\mathbb{F}unction}}$ , and

$$\widetilde{\mathcal{M}}\left(\widetilde{\mathfrak{M}}_{\lambda},\tilde{y}_{\mu}\right)=\alpha\int_{0}^{\left\{\frac{\widetilde{\delta_{S_m}}^{3}\left(\widetilde{\mathfrak{M}}_{\lambda,(f,\varphi)}\left(\widetilde{\widetilde{\mathfrak{M}}}_{\lambda}\right)\right)+\tilde{d}^{3}\left(\tilde{y}_{\mu,(f,\varphi)}(\widetilde{\mathfrak{N}}_{\mu})\right)}{1+\widetilde{\delta_{S_m}}^{2}\left(\widetilde{\mathfrak{M}}_{\lambda,(f,\varphi)}\left(\widetilde{\widetilde{\mathfrak{M}}}_{\lambda}\right)\right)+\tilde{d}^{2}\left(\tilde{y}_{\mu,(f,\varphi)}(\widetilde{\mathfrak{N}}_{\mu})\right)}\right\}}\xi(t)dt$$

$$+\gamma\int_{0}^{\{\tilde{d}(\widetilde{\mathfrak{M}}_{\lambda},\widetilde{\mathfrak{N}}_{\mu})\}}\xi(t)dt$$

Where $\tilde{\zeta},\widetilde{\gamma}\gtrsim\tilde{0}$ and $\tilde{2}\,\tilde{\zeta}+\widetilde{\gamma}\lesssim\tilde{1}$ is a $\mathbb{S}oft_{\ constant}$. Here $\xi$:$[\tilde{0},\ +\tilde{\infty})\ \rightarrow\ [\tilde{0},\ +\tilde{\infty})$ is a $\mathbb{L}esbesgue_{integrable}$ mapping which is summable on each compact subset of $[\tilde{0},+\tilde{\infty}),non_{negative}$ and $Such_{That}$ and $\forall\,\tilde{\in}\gtrsim\tilde{0},\int_{0}^{\in}\xi(t)dt\ \gtrsim\tilde{0},then$

Then $(f,\varphi)$ has a unique $\mathbb{S}oft_{\mathbb{F}ixed\ \mathbb{P}oint}$ in $\mathbb{SP}(\ \widetilde{\hat{W}})$ .

**"Results related to couple soft fixed point" is the title of chapter second.** The present part deals with some couple soft fixed are established for different type mappings.

**Theorem 2.1.1** : Let $(\widetilde{\hat{W}},\widetilde{\delta_{S_m}},\hat{E})$ $complete_{\,soft\ metric}$ where $\hat{E}\,is$ set of parameter , $\left(\widetilde{\hat{W}},\widetilde{\delta_{S_m}},\hat{E}\,,\tilde{\lesssim}\right)$ is a $Partial\ _{ordred\,set}$ , $\emptyset$: $\hat{E}\longrightarrow\hat{E}$, $\tilde{\mathfrak{S}}$ =( T,$\emptyset$) is

$\mathbb{S}oft\_\mathbb{C}ontinuous_{\ \mathbb{f}unction}$ and satisfy mix monotonic property:

$\forall\ \widetilde{\mathfrak{M}}_{a},\widetilde{\mathfrak{N}}_{b},\widetilde{\mathbb{U}_{a}},\widetilde{\mathbb{V}_{b}}$ in$\left(\widetilde{\hat{W}},\widetilde{\delta_{S_m}},\hat{E}\,,\tilde{\lesssim}\right)$ , $\tilde{\eta}\,\tilde{\in}\ \widetilde{(0}\,,\widetilde{1})$

$$\widetilde{\delta_{S_m}}\left(\tilde{\mathfrak{S}}\left(\widetilde{\mathfrak{M}}_{a},\widetilde{\mathfrak{N}}_{b}\right),\tilde{\mathfrak{S}}\left(\ \widetilde{\mathbb{U}_{a}},\widetilde{\mathbb{V}_{b}}\right)\right)$$

$$\tilde{\lesssim}\,\tilde{\eta}\,max\begin{Bmatrix}\left[\widetilde{\delta_{S_m}}\left(\widetilde{\mathfrak{M}}_{a},\tilde{\mathfrak{S}}\left(\ \widetilde{\mathbb{U}_{a}},\widetilde{\mathbb{V}_{b}}\right)+\widetilde{\delta_{S_m}}\left(\widetilde{\mathfrak{N}}_{b},\tilde{\mathfrak{S}}(\widetilde{\mathbb{V}_{b}},\widetilde{\mathbb{U}_{a}})\right)\right],\\ \left[\widetilde{\delta_{S_m}}\left(u_{a},\tilde{\mathfrak{S}}(\widetilde{\mathfrak{M}}_{a},\widetilde{\mathfrak{N}}_{b})\right)+\widetilde{\delta_{S_m}}\left(\mathbb{V}_{b},\tilde{\mathfrak{S}}(\widetilde{\mathfrak{N}}_{b},\widetilde{\mathfrak{M}}_{a})\right)\right]\end{Bmatrix}\qquad[2.1.1.\ (i)]$$

Then $\tilde{\mathfrak{S}}$ has $\mathbb{C}ouple_{\ \mathbb{S}oft\ \mathbb{F}ixed\ \mathbb{P}oint}$ in $\mathbb{S}oft_{\ \mathbb{M}etric\ \mathbb{S}pace}\left(\widetilde{\hat{W}},\widetilde{\delta_{S_m}},\hat{E}\,,\tilde{\lesssim}\right)$.

**Theorem 2.1.2**: Suppose $\left(\widetilde{\hat{W}},\widetilde{\delta_{S_m}},\hat{E}\,,\tilde{\lesssim}\right)$ is a $\mathbb{P}artial\ _{\mathbb{O}rdred\,\mathbb{S}et}$, $\widetilde{\delta_{S_m}}$: $\left(\widetilde{\hat{W}},\widetilde{\delta_{S_m}},\hat{E}\,,\tilde{\lesssim}\right)\times$ $\left(\widetilde{\hat{W}},\widetilde{\delta_{S_m}},\hat{E}\,,\tilde{\lesssim}\right)\longrightarrow$ R, be $soft\ _{metric}$ on $\widetilde{\hat{W}}$.

$\alpha$: $\left(\widetilde{\hat{W}},\widetilde{\delta_{S_m}},\hat{E}\,,\tilde{\lesssim}\right)\rightarrow\left(\widetilde{\hat{W}},\widetilde{\delta_{S_m}},\hat{E}\,,\tilde{\lesssim}\right)$ and

$\tilde{\mathfrak{S}}$: $\left(\widetilde{\hat{W}},\widetilde{\delta_{S_m}},\hat{E}\,,\tilde{\lesssim}\right)\times\left(\widetilde{\hat{W}},\widetilde{\delta_{S_m}},\hat{E}\,,\tilde{\lesssim}\right)\rightarrow\left(\widetilde{\hat{W}},\widetilde{\delta_{S_m}},\hat{E}\,,\tilde{\lesssim}\right)$, such that

$$\widetilde{\eth_{S_m}}\left(\widetilde{\mathfrak{S}}\left(\widetilde{\mathfrak{M}}_a,\widetilde{\mathfrak{N}}_b\right),\widetilde{\mathfrak{S}}\left(\widetilde{\mathfrak{M}}_a,\widetilde{\mathfrak{N}}_b\right)\right)+\widetilde{\eth_{S_m}}\left(\widetilde{\mathfrak{S}}\left(\widetilde{\mathfrak{N}}_b,\widetilde{\mathfrak{M}}_a\right),\widetilde{\mathfrak{S}}\left(\widetilde{v_b},\widetilde{u_a}\right)\right)$$

$$\widetilde{\leqq}\,\tilde{l}\left\{\widetilde{\eth_{S_m}}(\alpha\widetilde{\mathfrak{M}}_a,\alpha u_a)+\widetilde{\eth_{S_m}}(\alpha\widetilde{\mathfrak{N}}_b,\alpha v_b)\right\};\qquad [\ 2.1.1.\ (viii)\ ]$$

for some $\tilde{l}$ , $\tilde{0}\,\widetilde{\leqq}\,\tilde{l}\,\widetilde{<}\,\tilde{1}$and for all $\widetilde{\mathfrak{M}}_a,\widetilde{\mathfrak{N}}_b,\widetilde{\mathfrak{w}}_a$ , $\widetilde{\mathbb{v}_b}\ \widetilde{\in}\ \left(\widetilde{\hat{\mathrm{W}}},\widetilde{\eth_{S_m}},\hat{\mathrm{E}}\,,\widetilde{\leqq}\right)$

with $\alpha\widetilde{\mathfrak{M}}_a\geq\alpha\widetilde{\mathfrak{w}}_a$ , $\alpha\widetilde{\mathfrak{N}}_b\,\widetilde{\leqq}\,\alpha\widetilde{\mathbb{v}_b}$. A part from this new supposition

- $\widetilde{\mathfrak{S}}(\widetilde{\hat{\mathrm{W}}}\times\widetilde{\hat{\mathrm{W}}})\,\widetilde{\subseteq}\,\alpha(\widetilde{\hat{\mathrm{W}}})$
- $\alpha(\widetilde{\hat{\mathrm{W}}})$ is complete
- $\alpha$ $\mathbb{C}$ommutes with $\widetilde{\mathfrak{S}}$ and is $\mathbb{C}$ontinuous.$_{\mathbb{F}un}$
- The $\mathbb{M}_{ixed_{\text{alpha}_{\text{monotone}}}}$ rule on $\widetilde{\hat{\mathrm{W}}}$ is gratified by $\widetilde{\mathfrak{S}}$
- moreover $\widetilde{\mathfrak{S}}$ is either $\mathbb{C}ontinuous$ . or the Sequence. of $\widetilde{\hat{\mathrm{W}}}$ has following conditions
  - If a Seq. $\left\{\widetilde{\mathfrak{M}}_{a_n}^n\right\}$is non- decreasing $Such_{That}$
  - $\left\{\widetilde{\mathfrak{M}}_{a_n}^n\right\}\to\widetilde{\mathfrak{M}}_a\Rightarrow\left\{\widetilde{\mathfrak{M}}_{a_n}^n\right\}\leq\widetilde{\mathfrak{M}}_a$
  - if a Sequence $\left\{\widetilde{\mathfrak{N}}_{b_n}^n\right\}$is non- increasing, such that$\left\{\widetilde{\mathfrak{N}}_{b_n}^n\right\}\to\widetilde{\mathfrak{N}}_b$

    $\Rightarrow\left\{\widetilde{\mathfrak{N}}_{b_n}^n\right\}\leq\widetilde{\mathfrak{N}}_b$

  If $\exists\ \widetilde{\mathfrak{M}}_{a_0}^0,\widetilde{\mathfrak{N}}_{b_0}^0\,\widetilde{\in}\left(\widetilde{\hat{\mathrm{W}}},\widetilde{\eth_{S_m}},\hat{\mathrm{E}}\,,\widetilde{\leqq}\right)$ with $\alpha\widetilde{\mathfrak{M}}_{a_0}^0\leq\widetilde{\mathfrak{S}}\,(\widetilde{\mathfrak{M}}_{a_0}^0,\widetilde{\mathfrak{N}}_{b_0}^0)$

  And $\alpha\widetilde{\mathfrak{N}}_{b_0}^0\geq\widetilde{\mathfrak{S}}\,(\widetilde{\mathfrak{N}}_{b_0}^0,\widetilde{\mathfrak{M}}_{a_0}^0)$

$\Rightarrow$ $\widetilde{\mathfrak{S}}$ and α contain unique $\mathbb{C}ouple$ $\mathbb{S}$oft $\mathbb{F}ixed_{\,\mathbb{P}oint}$. That is there is unique $\widetilde{\mathfrak{M}}_a\,\widetilde{\in}\left(\widetilde{\hat{\mathrm{W}}},\widetilde{\eth_{S_m}},\hat{\mathrm{E}}\right)$ satisfies $\widetilde{\mathfrak{M}}_a=\widetilde{\mathfrak{S}}\,(\widetilde{\mathfrak{M}}_a,\widetilde{\mathfrak{M}}_a)=\alpha\widetilde{\mathfrak{M}}_a$

The chapter third of part first is intitled as **"Results related to Complete soft Usual Metric space, Soft Compact Metric spaces and Soft Banach spaces".** Further this chapter is subdivided in three parts.

**3.1 Fixed point Results on a Complete Soft Usual Metric Spaces**

The present work is related to the $\mathbb{F}ixed_{\,\mathbb{P}oint_{Theorems}}$ in $\mathbb{C}omplete_{\mathbb{S}oft}$ Usual_ $\mathbb{M}etric_{space}$ which are extension of well-known results on $complete_{\,metric}$ space as well as $\mathbb{C}omplete_{\mathbb{S}oft}$ $\mathbb{M}etric_{space}$. Main results are as follows:

**Theo. 3.1.1:** Let $(\mathrm{T},\S):\left(\widetilde{\mathrm{W}},|\ |,\mathrm{E}\right)\to\left(\widetilde{\mathrm{W}},|\ |,\mathrm{E}\right)$where $\left(\widetilde{\mathrm{W}},|,|,\mathrm{E}\right)$ is a $Soft_{usual\ _{Metric\ space}}$ if for all $\widetilde{\mathfrak{M}}_\lambda,\widetilde{\mathfrak{N}}_\mu\ \widetilde{\in}\ \mathbb{SP}(\widetilde{\mathrm{W}})$,

$$\tilde{|}\left((\mathrm{T},\S)(\widetilde{\mathfrak{M}}_\lambda)-\ (\mathrm{T},\S)(\widetilde{\mathfrak{N}}_\mu)\right)|\ \tilde{\leqq}$$

$$\tilde{1}/\tilde{2}\left[\left(\widetilde{\mathfrak{M}}_\lambda-(\mathrm{T},\S)(\widetilde{\mathfrak{N}}_\mu)\right)|+|\left(\widetilde{\mathfrak{N}}_\mu-((\mathrm{T},\S))(\widetilde{\mathfrak{M}}_\lambda)\right)|\right]$$

$$-\psi\left(|\left(\widetilde{\mathfrak{M}}_\lambda-(\mathrm{T},\S)(\widetilde{\mathfrak{N}}_\mu)\right)|,|\left(\widetilde{\mathfrak{N}}_\mu-(\mathrm{T},\S)(\widetilde{\mathfrak{M}}_\lambda)\right)|\right)\quad 3.1.1(i)$$

Where $\psi:\left[\tilde{0},\tilde{\infty}\right)^2\to[\tilde{0},\tilde{\infty})$ is a $continous_{mapping}\ Such_{tha}$

$$\psi(\widetilde{\mathfrak{M}}_\lambda,\widetilde{\mathfrak{N}}_\mu)=\ \widetilde{\widetilde{\mathfrak{M}}}_\lambda\Leftrightarrow\widetilde{\mathfrak{M}}_\lambda=\widetilde{\mathfrak{N}}_\mu=\ \tilde{0}.$$

$\Rightarrow\ (\mathrm{T},\S)$ has a $Unique_{Soft_{Fixed\ point}}$.

**Theorem 3.1.2.:** Let $(\mathrm{T},\S)):\left(\widetilde{\mathrm{W}},\widetilde{|.|},\mathrm{E}\right)\to\left(\widetilde{\mathrm{W}},\widetilde{|.|},\mathrm{E}\right)$ (where $\left(\widetilde{\mathrm{W}},|\ .\ |,\mathrm{E}\right)$ is a

$Complete_{Usual}\ Soft_{metric}$ space, be a $Complete_{Usual}\ Soft_{generalized}\ weak_{Contraction}$ that is the under condition is satisfied

if for all $\widetilde{\mathfrak{M}}_\lambda,\widetilde{\mathfrak{N}}_\mu\ \widetilde{\in}\ \mathbb{SP}(\ \widetilde{\mathrm{W}})$,

$$\left|\left((\mathrm{T},\S)(\widetilde{\mathfrak{M}}_\lambda)-(\mathrm{T},\S)(\widetilde{\mathfrak{N}}_\mu)\right)\right|\ \tilde{\leqq}$$

$$\widetilde{\mathcal{K}}\left[\max\begin{Bmatrix}\left|\left(\widetilde{\mathfrak{M}}_\lambda-((\mathrm{T},\S))\ (\widetilde{\mathfrak{M}}_\lambda)\right)\right|,|\left(\widetilde{\mathfrak{N}}_\mu,(\mathrm{T},\S)(\widetilde{\mathfrak{N}}_\mu)\right)|,\\ \left|\left(\widetilde{\mathfrak{M}}_\lambda,(\mathrm{T},\S)(\widetilde{\mathfrak{N}}_\mu)\right)\right|,|\left(\widetilde{\mathfrak{N}}_\mu,(\mathrm{T},\S)(\widetilde{\mathfrak{M}}_\lambda)\right),|(\widetilde{\mathfrak{M}}_\lambda,\widetilde{\mathfrak{N}}_\mu)|\end{Bmatrix}\right]$$

$$-\psi\left(\begin{matrix}|\left(\widetilde{\mathfrak{M}}_\lambda-(\mathrm{T},\S)(\widetilde{\mathfrak{M}}_\lambda)\right)|,|\left(\widetilde{\mathfrak{N}}_\mu-(\mathrm{T},\S)(\widetilde{\mathfrak{M}}_\mu)\right)|,|\left(\widetilde{\mathfrak{M}}_\lambda-(\mathrm{T},\S)(\widetilde{\mathfrak{N}}_\mu)\right)|,\\ |\left(\widetilde{\mathfrak{N}}_\mu-(\mathrm{T},\S)\ (\widetilde{\mathfrak{M}}_\lambda)\right)|,|(\widetilde{\mathfrak{M}}_\lambda,\widetilde{-\mathfrak{N}}_\mu|)\end{matrix}\right)$$

(

3.1.2(i))

Where $\widetilde{\mathcal{K}}\ \widetilde{\in}\left[\tilde{0},\ \tilde{1}\right),\ \psi:\left[\tilde{0},\tilde{\infty}\right)^5\to[\tilde{0},\tilde{\infty})$ is a continuous mapping:

$\psi(\widetilde{\mathfrak{M}}_1, \widetilde{\mathfrak{M}}_2, \widetilde{\mathfrak{M}}_3, \widetilde{\mathfrak{M}}_4, \widetilde{\mathfrak{M}}_5) = \tilde{0} \Leftrightarrow$one of $\widetilde{\mathfrak{M}}_1, \widetilde{\mathfrak{M}}_2, \widetilde{\mathfrak{M}}_3, \widetilde{\mathfrak{M}}_4, \widetilde{\mathfrak{M}}_5$ is equal to $\tilde{0}$ . Then $(\mathfrak{T}, \S)$has a $Unique_{\ Soft\ Fixed\ Point}$.

### 3.2 Common Fixed-Point Theorems Using Rational Contraction in Soft Compact Metric Spaces

**Theorem 3.2.1**. Let $\left(\widetilde{\mathrm{W}}, \widetilde{\eth_{S_{com}}}, \widetilde{\mathrm{E}}\right)$ be a $(\boldsymbol{SCMS})$and $\widetilde{\eth_{S_{com}}}$ be a metric on $\widetilde{\mathrm{W}}$, $Such_{That}$ $(\widetilde{\mathrm{W}}, \widetilde{\eth_{S_{com}}})$ is $complete_{soft_{metric}}$ and $(\widetilde{\mathfrak{F}}, \S), (\widetilde{\mathfrak{G}}, \S)$: $\widetilde{\mathrm{W}} \to \left(\widetilde{C(\mathrm{W})}\right)$

Satisfy :

1. $\widetilde{\mathfrak{U}_e} \in \widetilde{\mathrm{W}}, (\widetilde{\mathfrak{F}}, \S)(\widetilde{\mathfrak{U}_e}), (\widetilde{\mathfrak{G}}, \S)(\widetilde{\mathfrak{U}_e}) \cong \left(\widetilde{\mathrm{W}}\right)$,
2. $H\left((\widetilde{\mathfrak{F}}, \S)(\widetilde{\mathfrak{U}_e}), (\widetilde{\mathfrak{G}}, \S)(\widetilde{\mathfrak{N}_e})\right) \lesssim \psi_1 \begin{bmatrix} \widetilde{\eth_{S_{com}}}\left(\widetilde{\mathfrak{U}_e}, (\widetilde{\mathfrak{F}}, \S), (\widetilde{\mathfrak{U}_e})\right) \\ +\widetilde{\eth_{S_{com}}}(\widetilde{\mathfrak{N}_e}, (\widetilde{\mathfrak{G}}, \S)\widetilde{\mathfrak{N}_e}) \end{bmatrix}$

$$+\psi_2\left[\widetilde{\eth_{S_{com}}}(\widetilde{\mathfrak{U}_e}, (\widetilde{\mathfrak{G}}, \S)(\widetilde{\mathfrak{N}_e})) + \widetilde{\eth_{S_{com}}}(\widetilde{\mathfrak{N}_e}, (\widetilde{\mathfrak{F}}, \S)(\widetilde{\mathfrak{U}_e}))\right]$$

Where $\psi_1, \psi_2$ are non-negative real numbers and $\psi_1 + \psi_2 \lesssim 1$. Then $(\widetilde{\mathfrak{F}}, \S),\ and\ (\widetilde{\mathfrak{G}}, \S)$ has a $Common_{\ soft_{fixed\ point}}$

**Theorem 3.2.2**. Let $(\widetilde{\mathrm{W}}, \widetilde{\eth_{S_{com}}}, \tilde{E})$ be a $\mathbb{S}oft\ \mathbb{C}ompact_{\ \mathbb{M}etric_{spaces}}$ and $\widetilde{\eth_{S_{com}}}$ be a metric on $\widetilde{\mathrm{W}}$ such that $(\widetilde{\mathrm{W}}, \widetilde{\eth_{S_{com}}})$ is $\mathbb{C}omplete_{\mathbb{S}oft\ \mathbb{M}etric_{space}}$, and let mappings $(\widetilde{\mathfrak{F}}, \S), (\widetilde{\mathfrak{G}}, \S)$: $\widetilde{\mathrm{W}} \to \left(\widetilde{C\mathrm{W}}\right)$ $Such_{That}$

1. For each $\widetilde{\mathfrak{U}_e} \cong \widetilde{\mathrm{W}}, (\widetilde{\mathfrak{F}}, \S)(\widetilde{\mathfrak{U}_e}), (\widetilde{\mathfrak{G}}, \S)(\widetilde{\mathfrak{U}_e}) \cong$ closed set $\left(\widetilde{\mathrm{W}}\right)$,
2. $H\left((\widetilde{\mathfrak{F}}, \S)(\tilde{\alpha}), (\widetilde{\mathfrak{G}}, \S)(\tilde{\beta})\right) \leq \psi_1\left[\eth_{S_{com}}(\widetilde{\mathfrak{U}_e}, \tilde{\beta})) + \eth_{S_{com}}\left(\tilde{\beta}, (\widetilde{\mathfrak{G}}, \S)(\tilde{\beta})\right)\right]$
   $+\psi_2\left[\eth_{S_{com}}(\tilde{\beta}, (\widetilde{\mathfrak{G}}, \S)(\tilde{\beta})) + \eth_{S_{com}}\left(\widetilde{\mathfrak{U}_e}, T(\widetilde{\mathfrak{U}_e})\right)\right]$

$$+\psi_3\left[\eth_{S_{com}}(\tilde{\beta}, (\widetilde{\mathfrak{F}}, \S)(\widetilde{\mathfrak{U}_e})) + \eth_{S_{com}}(\widetilde{\mathfrak{U}_e}, (\widetilde{\mathfrak{G}}, \S)(\tilde{\beta}))\right]$$

where $\psi_1, \psi_2, \psi_3$ are non-negative real numbers and $\psi_1 + \psi_2 + \psi_3 < \frac{1}{3}$. Then$(\widetilde{\mathfrak{F}}, \S)\ and\ (\widetilde{\mathfrak{G}}, \S)$ has a common fixed point.

### 3.3. Unique Fixed-Point Theorems on soft Banach spaces

In this part some $\mathbb{U}nique_{\mathbb{F}ixed_{Point}}$ theorems in 2-soft $\mathbb{B}anach_{space}$ are proved which are motivated by Gahler and Das et.al.

**Theorem 3.3.1**: Let $(\widetilde{\hat{\mathrm{W}}}, \|\ \ \|, \widetilde{\hat{\mathrm{E}}})$ be a $\tilde{2}$- $\mathbb{S}oft_{\mathbb{B}anach\ \mathbb{S}pace}$ with $\widetilde{\hat{\mathrm{W}}} \geq \tilde{2}$. Let $(\widetilde{\mathfrak{F}}, \S), (\widetilde{\mathfrak{G}}, \S)$, be two soft $(\psi - \varphi)$ −contractive mapping from

$(\widetilde{\hat{\mathrm{W}}}, \|\ \ \|, \widetilde{\hat{\mathrm{E}}}) \longrightarrow (\widetilde{\hat{\mathrm{W}}}, \|\ \ \|, \widetilde{\hat{\mathrm{E}}})$ $Such_{That}$

$\|(\widetilde{\mathfrak{F}}, \S)\ \widetilde{\mathfrak{M}}_e - (\widetilde{\mathfrak{G}}, \S)\widetilde{\mathfrak{N}}_e, \widetilde{\mathfrak{w}}\| \lesssim$

$$(\psi - \varphi)\left[\begin{array}{c} \|\widetilde{\mathfrak{M}}_e - \widetilde{\mathfrak{N}}_e, \widetilde{\mathfrak{w}}\|, \\ \|\widetilde{\mathfrak{M}}_e - (\widetilde{\mathfrak{F}}, \S), \widetilde{\mathfrak{M}}_e, \widetilde{\mathfrak{w}}\|, \\ \left\|\widetilde{\mathfrak{N}}_e - \widetilde{(\mathfrak{G}, \S)}\ \widetilde{\mathfrak{N}}_e \widetilde{\mathfrak{w}}\right\|, \\ \dfrac{\left\|\widetilde{\mathfrak{M}}_e - \widetilde{(\mathfrak{G}, \S)}\ \widetilde{\mathfrak{N}}_e, \widetilde{\mathfrak{w}}\right\| + \|\widetilde{\mathfrak{N}}_e - (\widetilde{\mathfrak{F}}, \S)\ \widetilde{\mathfrak{M}}_e, \widetilde{\mathfrak{w}}\|}{2} \end{array}\right]$$

For all $\widetilde{\mathfrak{M}}_e, \widetilde{\mathfrak{N}}_e, \widetilde{\mathfrak{w}} \in \widetilde{\hat{\mathrm{W}}}$.. Then $(\widetilde{\mathfrak{F}}, \S)$ *and* $(\widetilde{\mathfrak{G}, \S})$ have a $unique_{\ common\ _{soft\ fixed}}$ point in $\widetilde{\hat{\mathrm{W}}}$.

**Corollary 3.3.2**. Let $(\widetilde{\hat{\mathrm{W}}}, \|\ \ \|, \widetilde{\hat{\mathrm{E}}})$ be a $2 - Soft_{Banach_{Spaces}}$

with $\widetilde{\hat{\mathrm{W}}} \geq 2$. Let $(\widetilde{\mathfrak{G}}, \S)$, be soft $(\psi - \varphi) - contractive_{mapping}$, from

$(\widetilde{\hat{\mathrm{W}}}, \|\ \ \|, \widetilde{\hat{\mathrm{E}}}) \longrightarrow (\widetilde{\hat{\mathrm{W}}}, \|\ \ \|, \widetilde{\hat{\mathrm{E}}})$ $Such_{That}$

$\|(\widetilde{\mathfrak{G}, \S})\ \widetilde{\mathfrak{M}}_e - (\mathfrak{G}, \S)\widetilde{\mathfrak{N}}_e, \widetilde{\mathfrak{w}}\|$

$$\lesssim (\psi - \varphi)\left[\begin{array}{c} \|\widetilde{\mathfrak{M}}_e - \widetilde{\mathfrak{N}}_e, \widetilde{\mathfrak{w}}\|, \\ \left\|\widetilde{\mathfrak{M}}_e - \widetilde{(\mathfrak{G}, \S)}\ \mathfrak{M}_e, \widetilde{\mathfrak{w}}\right\|, \\ \left\|\widetilde{\mathfrak{N}}_e - \widetilde{(\mathfrak{G}, \S)}\ \widetilde{\mathfrak{N}}_e, \widetilde{\mathfrak{w}}\right\|, \\ \dfrac{\left\|\widetilde{\mathfrak{M}}_e - (\widetilde{\mathfrak{G}, \S})\widetilde{\mathfrak{N}}_e, \widetilde{\mathfrak{w}}\right\| + \|\widetilde{\mathfrak{N}}_e - (\widetilde{\mathfrak{G}, \S})\widetilde{\mathfrak{M}}_e, \widetilde{\mathfrak{w}}\|}{2} \end{array}\right]$$

For all $\widetilde{\mathfrak{M}}_e, \widetilde{\mathfrak{N}}_e, \widetilde{\omega} \in \widetilde{\hat{W}}$. Then $(\widetilde{\mathfrak{G}}, \S)$, have a $unique_{softfixed_{point}}$ in $\widetilde{\hat{W}}$

**Theorem 3.3.3.** Let $(\widetilde{\hat{W}}, \|\ \|, \widetilde{\hat{E}})$ be a $2-Soft_{Bana\ \ Spaces}$ with $\widetilde{\hat{W}} \geq 2$. Let $(\widetilde{\mathfrak{G}}, \S), (\widetilde{\mathfrak{G}}, \S)$ be soft $(\psi-\varphi)-contractive_{mapping}$, from

$(\widetilde{\hat{W}}, \|\ \|, \widetilde{\hat{E}}) \longrightarrow (\widetilde{\hat{W}}, \|\ \|, \widetilde{\hat{E}})$ $Such_{That}$

$$\left\|(\widetilde{\mathfrak{F}, \S})\,\mathfrak{M}_e - (\widetilde{\mathfrak{G}, \S})\,\widetilde{\mathfrak{N}}_e, \widetilde{\omega}\right\| \leq (\psi-\varphi)\,\widetilde{\mathbb{k}}\,\|\widetilde{\mathfrak{M}}_e - \widetilde{\mathfrak{N}}_e, \widetilde{\omega}\|$$

For all $\widetilde{\mathfrak{M}}_e, \widetilde{\mathfrak{N}}_e, \widetilde{\omega} \in \widetilde{\hat{W}}, 0 < \widetilde{\mathbb{k}} < 1$.

Then $(\widetilde{\mathfrak{F}}, \S)$ and $(\widetilde{\mathfrak{G}}, \S)$have a unique $common_{fixed\ point}$ in $\widetilde{\hat{W}}$.

**Theorem 3.3.4.** $(\widetilde{\hat{W}}, \|\ \|, \widetilde{\hat{E}})$ be a $2-Soft_{Banach_{Spaces}}$ with $\widetilde{\hat{W}} \geq 2$. Let $(\widetilde{\mathfrak{G}}, \S)$, be soft $(\psi-\varphi)-contractive_{mapping}$, from

$(\widetilde{\hat{W}}, \|\ \|, \widetilde{\hat{E}}) \longrightarrow (\widetilde{\hat{W}}, \|\ \|, \widetilde{\hat{E}})$ $Such_{That}$

$$\left\|(\widetilde{\mathfrak{F}, \S})\mathfrak{M}_e - (\widetilde{\mathfrak{G}, \S})\widetilde{\mathfrak{N}}_e, \widetilde{\omega}\right\| \leq (\psi-\varphi)\,\widetilde{\mathbb{k}}\left[\left\|\widetilde{\mathfrak{M}}_e - (\widetilde{\mathfrak{F}, \S})\widetilde{\mathfrak{N}}_e, \widetilde{\omega}\right\| + \left\|\widetilde{\mathfrak{N}}_e - (\widetilde{\mathfrak{G}, \S})\mathfrak{M}_e, \widetilde{\omega}\right\|\right]$$

For all $\widetilde{\mathfrak{M}}_e, \widetilde{\mathfrak{N}}_e, \widetilde{\omega} \in \widetilde{\hat{W}}, 0 < \widetilde{\mathbb{k}} < 1$.

Then $(\widetilde{\mathfrak{F}}, \S)$ and $(\widetilde{\mathfrak{G}}, \S)$have a unique $common_{fixed\ point}$ in $\widetilde{\hat{W}}$.

**Theorem 3.3.5.** $(\widetilde{\hat{W}}, \|\ \|, \widetilde{\hat{E}})$ be a $2-Soft_{Ban\ \ Spaces}$ with $\widetilde{\hat{W}} \geq 2$. Let $(\widetilde{\mathfrak{G}}, \S), (\widetilde{\mathfrak{G}}, \S)$ be soft $(\psi-\varphi)-contractive_{mapping}$, from

$(\widetilde{\hat{W}}, \|\ \|, \widetilde{\hat{E}}) \longrightarrow (\widetilde{\hat{W}}, \|\ \|, \widetilde{\hat{E}})$ $Such_{That}$

$$\left\|(\widetilde{\mathfrak{F}, \S})\mathfrak{M}_e - (\widetilde{\mathfrak{G}, \S})\widetilde{\mathfrak{N}}_e, \widetilde{\omega}\right\|$$

$$\widetilde{\leqq}(\psi-\varphi)\begin{bmatrix}\|\tilde{x}-\tilde{y},\tilde{\omega}\|,\\ \dfrac{\left\|\widetilde{\mathfrak{M}_e}-\widetilde{(\mathfrak{F},\S)}\mathfrak{M}_e,\tilde{\omega}\right\|\ \left\|\widetilde{\mathfrak{N}_e}-\widetilde{(\mathfrak{G},\ \S)}\widetilde{\mathfrak{N}_e},\ \tilde{\omega}\right\|}{2},\\ 0,\\ \left\|\widetilde{\mathfrak{M}_e}-\widetilde{(\mathfrak{G},\ \S)}\widetilde{\mathfrak{N}_e},\tilde{\omega}\right\|+\left\|\widetilde{\mathfrak{N}_e}-\widetilde{(\mathfrak{F},\S)}\mathfrak{M}_e,\tilde{\omega}\right\|\end{bmatrix}$$

For all $\widetilde{\mathfrak{M}_e},\widetilde{\mathfrak{N}_e},\tilde{\omega}\in\tilde{\hat{W}},0<\tilde{\mathbb{k}}<1$. Then $(\widetilde{\mathfrak{F},\S})$ and $(\widetilde{\mathfrak{G},\ \S})$have a unique $Soft\_common_{fixed\ point}$ in $\tilde{\hat{W}}$.

**Corollary 3.3.6**. $(\widetilde{\hat{W}},\|\ \ \|,\widetilde{\hat{E}})$ be a $2-Soft_{Banach_{Spaces}}$ with $\widetilde{\hat{W}}\geq 2$. Let $(\widetilde{\mathfrak{G},\ \S})$, be soft $(\psi-\varphi)-contractive_{mapping}$, from

$(\widetilde{\hat{W}},\|\ \ \|,\widetilde{\hat{E}})\longrightarrow(\widetilde{\hat{W}},\|\ \ \|,\widetilde{\hat{E}})\quad Such_{That}$

$$\left\|\widetilde{(\mathfrak{G},\ \S)}\mathfrak{M}_e-\widetilde{(\mathfrak{G},\ \S)}\tilde{y},\tilde{\omega}\right\|$$

$$\widetilde{\leqq}(\psi-\varphi)\begin{bmatrix}\|\widetilde{\mathfrak{M}_e}-\widetilde{\mathfrak{N}_e},\tilde{\omega}\|,\\ \dfrac{\|\widetilde{\mathfrak{M}_e}\ -\widetilde{(\mathfrak{G},\ \S)}\widetilde{\mathfrak{M}_e},\tilde{\omega}\|+\ \left\|\tilde{y}-\widetilde{(\mathfrak{G},\ \S)}\widetilde{\mathfrak{N}_e},\tilde{\omega}\right\|}{2},\\ \left\|\widetilde{\mathfrak{M}_e}-\widetilde{(\mathfrak{G},\ \S)}\widetilde{\mathfrak{N}_e,\tilde{\omega}}\right\|+\left\|\tilde{y}-\widetilde{(\mathfrak{G},\ \S)}\mathfrak{M}_e,\tilde{\omega}\right\|\end{bmatrix}$$

For all $\widetilde{\mathfrak{M}_e},\widetilde{\mathfrak{N}_e},\tilde{\omega}\in\tilde{\hat{W}}$. Then $(\widetilde{\mathfrak{G},\ \S})$ has a $Unique_{soft_{fixed point}}$ in $\tilde{\hat{W}}$.

**Theorem 3.3.7**. Let $(\widetilde{\hat{W}},\|\ \ \|,\widetilde{\hat{E}})$ be a $2-$soft Banach space

with $\widetilde{\hat{W}}\geq 2$. Let $(\widetilde{\mathfrak{F},\S}),(\widetilde{\mathfrak{G},\ \S})$, be two soft $(\psi-\varphi)-$contractive mapping from

$(\widetilde{\hat{W}},\|\ \ \|,\widetilde{\hat{E}})\longrightarrow(\widetilde{\hat{W}},\|\ \ \|,\widetilde{\hat{E}})\quad Such_{That}$

$$\|\widetilde{(\mathfrak{F},\S)}\ \widetilde{\mathfrak{M}_e}-\widetilde{(\mathfrak{G},\ \S)}\widetilde{\mathfrak{N}_e},\tilde{\omega}\|\leq(\psi-\varphi)\tilde{\mathbb{k}}\begin{bmatrix}\|\widetilde{\mathfrak{M}_e}-\ \widetilde{(\mathfrak{F},\S)}\widetilde{\mathfrak{N}_e},\tilde{\omega}\|\\ +\|\tilde{y}-\widetilde{(\mathfrak{G},\ \S)}\widetilde{\mathfrak{M}_e},\tilde{\omega}\|\end{bmatrix}$$

For all $\widetilde{\mathfrak{M}_e},\widetilde{\mathfrak{N}_e},\tilde{\omega}\in\hat{W},0<\lambda_2<\frac{1}{2}$. Then $\widetilde{(\mathfrak{F},\S)}\ and\ \widetilde{(\mathfrak{G},\ \S)}$ have a $unique_{common_{fixedepoint}}$ in $\widetilde{\hat{W}}$.

**"Some fixed point theorems in the context of Rough"** is the title of part second. Further this part is divided in three subparts as Rough metric spaces, compact rough metric space and Rough Banach spaces.

**Theorem 4.1.1**.: Let < Æ (CƆ), $ð_{R_{ms}}$ > be a $rough_{complete\ metric\ space}$. with respect to $Approximatin\ _{space}$, $Å_{𝕊}$ = (𝔜, $\bar{R}$) and Ĥ: (CƆ)⟶ (CƆ)

$$ð_{R_{ms}}(Ĥu, Ĥv) \leq ς\left[\frac{ð_{R_{ms}}{}^{3}(u, Ĥu) + ð_{R_{ms}}{}^{3}(v, Ĥv)}{1 + ð_{R_{ms}}{}^{2}(u, Ĥu) + ð_{R_{ms}}{}^{2}(v, Ĥv)}\right]$$

$$+ б\left[\frac{ð_{R_{ms}}{}^{3}(u,Ĥv)+ð_{R_{ms}}{}^{3}(v,Ĥu)}{1+ð_{R_{ms}}{}^{2}(u,Ĥv)+ð_{R_{ms}}{}^{2}(v,Ĥu)}\right]$$

$$+ ρ\max\left[ð_{R_{ms}}(u, v), \frac{ð_{R_{ms}}(v,Ĥu)}{1+ð_{R_{ms}}(u,Ĥv)ð_{R_{ms}}(v,Ĥu)ð_{R_{ms}}(u,v)}\right]$$

$$+ y\, ð_{R_{ms}}(u, v)$$

$\forall\ u, v \underbrace{\in_{Å_{𝕊}}} CƆ$, $0 \leq ς, б, y, ρ < 1$, $2ς + 2б + y + ρ < 1$, (4.1.1 (i))

And, for some $u_0 \underbrace{\in_{Å_{𝕊}}} CƆ$, the rough sequence of iterates $\{Ĥ^n(u_0)\}$ has a

rough subsequence$\{Ĥ^{nk}(u_0)\}$ with $z = \lim_{n\to\infty}\{Ĥ^{nk}(u_0)\}$ ----( 4.1.1.(ii))

then $z$ is a $unique_{\ roug\ \ fixed\ point}$ of Ĥ with respect to rough set with respect to $Approximatin\ _{space}$, $Å_{𝕊}$ = (𝔜, $\bar{R}$).

**Corollary4.1.2:** Let < Æ (CƆ), $ð_{R_{ms}}$ > be a $rough_{complete\ metric\ space}$. with respect to $approximation\ _{space}$ $Å_{𝕊}$ and Ĥ: CƆ⟶ CƆ

$$ð_{R_{ms}}(Ĥu, Ĥv) \leq ς\left[\frac{ð_{R_{ms}}{}^{3}(u, Ĥu) + ð_{R_{ms}}{}^{3}(v, Ĥv)}{1 + ð_{R_{ms}}{}^{2}(u, Ĥu) + ð_{R_{ms}}{}^{2}(v, Ĥv)}\right]$$

$$+ б\left[\frac{ð_{R_{ms}}{}^{3}(u,Ĥv)+ð_{R_{ms}}{}^{3}(v,Ĥu)}{1+ð_{R_{ms}}{}^{2}(u,Ĥv)+ð_{R_{ms}}{}^{2}(v,Ĥu)}\right]$$

$$+ ρ\max\left[ð_{R_{ms}}(u, v), \frac{ð_{R_{ms}}(v,Ĥu)}{1+ð_{R_{ms}}(u,Ĥv)ð_{R_{ms}}(v,Ĥu)ð_{R_{ms}}(u,v)}\right]$$

$$+ \gamma\, \eth_{R_{ms}}(u, v)$$

$$+ \delta\ [\ \eth_{R_{ms}}(u, \check{H}u) + \eth_{R_{ms}}(v, \check{H}v\ )]$$

$$+ \eta\ [\ \eth_{R_{ms}}(u, \check{H}v) + \eth_{R_{ms}}(v, \check{H}u\ )]$$

$\forall\ \ u, v \in_{\underbrace{\text{Å}_{\S}}}$ CƆ , $0 \le \varsigma, \text{ɓ}, \gamma, \rho, \delta, \eta\ < 1,$

$2\varsigma + 2\,\text{ɓ} + 2\,\eta + 2\,\delta + \gamma + \rho\ < 1$ ----( 4.1.2 (i))

And, for some $u_0 \in_{S_{Æ}}$ CƆ , the rough sequence of iterates $\{\check{H}^{n}(u_0)\}$ has a rough subsequence$\{\check{H}^{nk}(u_0)\}$ with $z = \lim_{n\to\infty}\{\check{H}^{nk}(u_0)\}$ ----( 4.1.2. (ii) )

then z is a $unique_{\ rough\ fixed\ point}$ of Ĥ with respect to rough set.

**THEOREM 4.1.3:** Let < Æ (CƆ), $\eth_{R_{ms}} >$ be a $rough_{complete\ metric\ space}$. with respect to $approximation_{\ space}$ $\text{Å}_{\S}$ and Ĥ: Æ (CƆ)⟶ Æ (CƆ), for any pair of points u, v belonging to an everywhere dense subset ℚ of CƆ

$$\eth_{R_{ms}}(\check{H}u, \check{H}v) \le \varsigma\left[\frac{\eth_{R_{ms}}{}^{3}(u, \check{H}u) + \eth_{R_{ms}}{}^{3}(v, \check{H}v)}{1 + \eth_{R_{ms}}{}^{2}(u, \check{H}u) + \eth_{R_{ms}}{}^{2}(v, \check{H}v)}\right]$$

$$+\text{ɓ}\left[\frac{\eth_{R_{ms}}{}^{2}(u,\check{H}v)+\eth_{R_{ms}}{}^{2}(v,\check{H}u)}{1+\eth_{R_{ms}}(u,\check{H}v)+\eth_{R_{ms}}(v,\check{H}u)}\right]$$

$$+ \rho\max\left[\eth_{R_{ms}}(u, v), \frac{\eth_{R_{ms}}(v,\check{H}u\ )}{1+\eth_{R_{ms}}(u,\check{H}v)\eth_{R_{ms}}(v,\check{H}u\ )\eth_{R_{ms}}(u,v)}\ \right]$$

$$+ \gamma\, \eth_{R_{ms}}(u, v)$$

$$+ \delta\ [\ \eth_{R_{ms}}(u, \check{H}u) + \eth_{R_{ms}}(v, \check{H}v\ )]$$

$$+ \eta\ [\ \eth_{R_{ms}}(u, \check{H}v) + \eth_{R_{ms}}(v, \check{H}u\ )]$$

$\forall\ \ \forall\ \ u, v \in_{\underbrace{\text{Å}_{\S}}}$ CƆ, $0 \le \varsigma, \text{ɓ}, \gamma, \rho, \delta, \eta\ < 1,$

$2\varsigma + 2\,\text{ɓ} + 2\,\eta + 2\,\delta + \gamma + \rho\ < 1$ ----( 4.1.3 (i))

And, for some $[u_0]_{\bar{R}} \underbrace{\in_{Å_{\mathbb{S}}}} C\mathfrak{O}$ , the rough sequence of iterates $\{\check{H}^n(u_0)\}$ has a rough subsequence $\{\check{H}^{nk}(u_0)\}$ with $z = \lim_{n\to\infty}\{\check{H}^{nk}(u_0)\}$ ----( 4.1.3). (ii) )

then $z$ is a $unique_{\ rough_{\ fixed_{\ point}}}$ of $\check{H}$ with respect to rough set through approximation space.

**THEOREM 4.1.4**:- Let < Æ (C$\mathfrak{O}$), $ð_{R_{ms}} >$ be a $rough_{complete\ metric_{\ space}}$. with respect to $approximation_{\ space}$ $Å_{\mathbb{S}}$ and $\check{H}$: Æ (C$\mathfrak{O}$)⟶ Æ (C$\mathfrak{O}$), rough converging point wise to $\check{H}$ and let

$$ð_{R_{ms}}(\check{H}_i u, \check{H}_i v) \le \varsigma\left[\frac{ð_{R_{ms}}{}^3(u, \check{H}_i u) + ð_{R_{ms}}{}^3(v, \check{H}_i v)}{1 + ð_{R_{ms}}{}^2(u, \check{H}_i u) + ð_{R_{ms}}{}^2(v, \check{H}_i v)}\right]$$

$$+ ƃ\left[\frac{ð_{R_{ms}}{}^2(u,\check{H}_i v)+ð_{R_{ms}}{}^2(v,\check{H}_i u)}{1+ð_{R_{ms}}(u,\check{H}_i v)+ð_{R_{ms}}(v,\check{H}_i u)}\right]$$

$$+ \rho\max\left[ð_{R_{ms}}(u, v), \frac{ð_{R_{ms}}(v,\check{H}_i v)}{1+ð_{R_{ms}}(u,\check{H}_i v)ð_{R_{ms}}(v,\check{H}_i u)ð_{R_{ms}}(u,v)}\right]$$

$$+ \gamma\, ð_{R_{ms}}(u, v)$$

$\forall\ u, v \underbrace{\in_{Å_{\mathbb{S}}}} C\mathfrak{O}$ , $0 \le \varsigma, ƃ, \gamma, \rho < 1$, $2\varsigma + 2ƃ + \gamma + \rho < 1$ --( 3.1.4 (i))

i =1,2,3- for some $u_0 \underbrace{\in_{Å_{\mathbb{S}}}} C\mathfrak{O}$ , the rough sequence of iterates $\{\check{H}^n(u_0)\}$ has a rough subsequence $\{\check{H}^{nk}(u_0)\}$

with $z = \lim_{n\to\infty}\{\check{H}^{nk}(u_0)\}$ ----( 4.1.4.(ii))

If each $\check{H}_i$ has a $fixed_{point}$ $z_i$ and $\check{H}$ has a rough $fixed_{point}$ z, then the sequence $\{z_i\}$ converges to z.

**4.2 fixed point theorems in compact rough metric spaces**

**Theorem 4.2.1:** Let $Å_{\mathbb{S}}$ = ( $\mathfrak{U}$ , $\bar{R}$ ), be an approximation space and ( $\mathcal{C}$, $ð_{R_{ms}}$) be a $compact_{rough_{metric}}$ space, and let $\mathcal{H}$ be a continuous self-mapping on C which satisfies the inequality:

$ð_{R_{ms}}(\mathcal{H}[u]_R, \mathcal{H}[v]_R) <$

$$\varsigma_1\{\frac{\eth_{R_{ms}}([u]_R,\mathcal{H}[v]_R).\eth_{R_{ms}}([v]_R,\mathcal{H}[u]_R).\ \eth_{R_{ms}}([u]_R,\mathcal{H}[u]_R)+\eth_{R_{ms}}([v]_R,\mathcal{H}[v]_R)+\eth_{R_{ms}}([u]_R,[v]_R).}{1+\eth_{R_{ms}}([u]_R,\mathcal{H}[v]_R).\eth_{R_{ms}}([v]_R,\mathcal{H}[u]_R)\eth_{R_{ms}}([u]_R,\mathcal{H}[u]_R)\eth_{R_{ms}}([v]_R,\mathcal{H}[v]_R)\eth_{R_{ms}}([u]_R,[v]_R)}$$

$$+\varsigma_2\{\eth_{R_{ms}}([v]_R,\mathcal{H}[u]_R)+\eth_{R_{ms}}([v]_R,\mathcal{H}[v]_R)\}+\varsigma_3\{\eth_{R_{ms}}([u]_R,\ [v]_R)\}$$

$$+\max\varsigma_4\ \{\eth_{R_{ms}}([u]_R,\ [v]_R),\eth_{R_{ms}}([u]_R,\mathcal{H}[u]_R),$$

$$\eth_{R_{ms}}([v]_R,\ \mathcal{H}[v]_R),\frac{1}{2}\ [\eth_{R_{ms}}([u]_R,\mathcal{H}[v]_R)+\ \eth_{R_{ms}}([v]_R,\mathcal{H}[u]_R).\},$$

Where $[v]_R,[u]_R\ \underbrace{\in_{S_{\mathring{A}_{\mathbb{S}}}}}\ \boldsymbol{\mathcal{C}}$ with approximation space $\mathring{A}_{\mathbb{S}}$ and $\varsigma_i$ are non-negative rough real numbers such that $2\varsigma_1+\varsigma_2+\varsigma_3+\varsigma_4<1$, then $\mathcal{H}$ admits a unique rough fixed point in $\boldsymbol{\mathcal{C}}$

**Theorem 4.2.2:** Let $\mathring{A}_{\mathbb{S}}$ = ( $\mathfrak{U}$ , $\bar{R}$ ), be an approximation space and ( $\boldsymbol{\mathcal{C}}$, $\eth_{R_{ms}}$) be a $compact_{rough_{metric}}$ space, and let $\mathcal{H}$ be a continuous self-mapping on $\boldsymbol{\mathcal{C}}$ which satisfies the inequality:

$$\eth_{R_{ms}}(\mathcal{H}[u]_R,\mathcal{H}[v]_R)<$$

$$\varsigma_1\{\frac{\eth_{R_{ms}}([u]_R,\mathcal{H}[v]_R).\eth_{R_{ms}}([v]_R,\mathcal{H}[u]_R).\ \eth_{R_{ms}}([u]_R,\mathcal{H}[u]_R)+\eth_{R_{ms}}([u]_R,[v]_R).}{1+\eth_{R_{ms}}([u]_R,\mathcal{H}[v]_R).\eth_{R_{ms}}([v]_R,\mathcal{H}[u]_R)\eth_{R_{ms}}([u]_R,\mathcal{H}[u]_R)\eth_{R_{ms}}([v]_R,\mathcal{H}[v]_R)\eth_{R_{ms}}([u]_R,[v]_R)}$$

$$+\varsigma_2\{\eth_{R_{ms}}([v]_R,\mathcal{H}[u]_R)+\eth_{R_{ms}}([v]_R,\mathcal{H}[v]_R)\}+\varsigma_3\{\eth_{R_{ms}}([u]_R,\ [v]_R)\}$$

Where $[u]_R,[v]_R\ \underbrace{\in_{S_{Æ}}}\ C$ with approximation space $\mathring{A}_{\mathbb{S}}$ and $\varsigma_i$ are non-negative rough real numbers such that $2\varsigma_1+\varsigma_2+\varsigma_3<1$, then $\mathcal{H}$ admits a unique rough fixed point in C.

**4.3 fixed point theorems in rough Banach spaces**

Present work is concerning with $fixed_{\ point_{theorems}}$ in $Rough_{\ Ban\ \ space}$ for rational type mappings. First the concept of $Rough_{\ Ban\ \ space}$ ( RBs) along with some basic properties are established . Next some $fixed_{\ point_{theorems}}$ and $common-fixed_{\ point_{theorems}}$ are proved. The established results are generalization forms of some well-known results which will be useful to solve engineering problems as well as useful in decision making phenomena.

**Theorem 4.3.1 :** Let (Æ,(CƆ), $\left|\left|.\right|\right|_{\bar{R}}$) be a $Rough_{Banac\ \ space}$ in $Approximation_{\ space}$

( $\mathfrak{Y}$, $\bar{\mathrm{R}}$) where $\mathfrak{Y} \neq \emptyset$, and $\bar{\mathrm{R}}$ is an equivalence relation. $\mathfrak{T}$: Ↄ → Ↄ :

(4.3.1.(i)) $\mathfrak{T}^2 = \mathrm{I}$, where I is $Identity_{\ mapping}$

(4.3.1.(ii)) $\left|\left|\mathfrak{T}(\mathfrak{M}) - \mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}$

$$\leq \varsigma \frac{\left|\left|\mathfrak{M} - \mathfrak{T}\mathfrak{M}\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{N} - \mathfrak{T}\mathfrak{M}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{N} - \mathfrak{T}\mathfrak{N}\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{M} - \mathfrak{T}\mathfrak{N}\right|\right|_{\bar{\mathrm{R}}}}{1 + \left|\left|\mathfrak{M} - \mathfrak{T}\mathfrak{M}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{N} - \mathfrak{T}\mathfrak{M}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{N} - \mathfrak{T}\mathfrak{N}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{M} - \mathfrak{T}\mathfrak{N}\right|\right|_{\bar{\mathrm{R}}}}$$

$$+\beta \frac{\left|\left|\mathfrak{M} - \mathfrak{T}\mathfrak{M}\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{N} - \mathfrak{T}\mathfrak{N}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{M} - \mathfrak{T}\mathfrak{N}\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{N} - \mathfrak{T}\mathfrak{M}\right|\right|_{\bar{\mathrm{R}}}}{1 + \left|\left|\mathfrak{M} - \mathfrak{T}\mathfrak{M}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{N} - \mathfrak{T}\mathfrak{N}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{M} - \mathfrak{T}\mathfrak{N}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{N} - \mathfrak{T}\mathfrak{M}\right|\right|_{\bar{\mathrm{R}}}}$$

$$+\sigma\left|\left|\mathfrak{M} - \mathfrak{N}\right|\right|_{\bar{\mathrm{R}}}$$

$$\forall \ \left|\left|\mathfrak{M} - \mathfrak{T}\mathfrak{M}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{N} - \mathfrak{T}\mathfrak{M}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{N} - \mathfrak{T}\mathfrak{N}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{M} - \mathfrak{T}\mathfrak{N}\right|\right|_{\bar{\mathrm{R}}} \neq 0$$

$$\left|\left|\mathfrak{M} - \mathfrak{T}\mathfrak{M}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{N} - \mathfrak{T}\mathfrak{N}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{M} - \mathfrak{T}\mathfrak{N}\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{N} - \mathfrak{T}\mathfrak{M}\right|\right|_{\bar{\mathrm{R}}} \neq 0$$

$\forall\, \mathfrak{M}, \mathfrak{N} \underbrace{\in_{\text{Å}_{\text{Ş}}}} \text{Æ}, (\text{Ↄ}), 0 \leq \varsigma, \beta, \ \sigma, \ \ \mathfrak{M} \neq \mathfrak{N}$ , then $\mathfrak{T}$ has a $rough_{fixed\ point}$

if $8\varsigma + 7\,\beta + 4\sigma < 8$, *also if,* $2\beta + \ \sigma < 2$ *then* $\mathfrak{T}$ *has unique* $rough_{fixed\ point}$

**Theorem 4.3.2**: Let (Æ,(Ↄ), $\left|\left|.\right|\right|_{\bar{\mathrm{R}}}$) be a $Rough_{Ban\ \ \ space}$ in $Approximation\ _{space}$ ( $\mathfrak{Y}$, $\bar{\mathrm{R}}$) where $\mathfrak{Y} \neq \emptyset$, and $\bar{\mathrm{R}}$ is an equivalence relation.

L be closed and $convex\ _{subset}$ of Æ,(Ↄ)

$\mathfrak{T}$: L → L, and $\mathfrak{Q}$ : L → L, satisfying the following conditions

(4.3.2.(i)) $\mathfrak{T}^2 = \mathrm{I}$, and $\mathfrak{Q}^2 = \mathrm{I}$ where I is $Identity\ _{mapping}$

4.3.2.(ii )) $\mathfrak{T}\mathfrak{Q} = \mathfrak{Q}\mathfrak{T}$

4.3.2.(iii)) $\left|\left|\mathfrak{T}(\mathfrak{M}) - \mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}$

$$\leq \varsigma \frac{\left|\left|\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}}{1+\left|\left|\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}}$$

$$+\mathcal{B}\frac{\left|\left|\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}}{1+\left|\left|\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}}$$

$$+\sigma\left|\left|\mathfrak{Q}\mathfrak{M}-\mathfrak{Q}\mathfrak{N}\right|\right|_{\bar{\mathrm{R}}}$$

$$\forall$$

$$\left|\left|\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}\neq 0$$

$$\left|\left|\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}\neq 0$$

$\forall\, \mathfrak{M},\mathfrak{N} \underbrace{\in_{\text{Å}_{\$}}} \text{Æ},(\text{CƆ}), 0 \leq \varsigma, \mathcal{B}, \sigma,\ \mathfrak{M}\neq\mathfrak{N}$, then $\mathfrak{T}$ *and* $\mathfrak{Q}$ have common $rough_{fixed\ point}$ if $8\varsigma+7\,\mathcal{B}+4\sigma<8$, *also if,* $2\mathcal{B}+\sigma<2$ *then* $\mathfrak{T}$ *and* $\mathfrak{Q}$ have unique $rough_{fixed\ point}$

**Theorem 4.3.3**: Let (Æ,(CƆ), $\left|\left|.\right|\right|_{\bar{\mathrm{R}}}$) be a $Rough_{Banach\ space}$ in $Approximation\ _{space}$ ( $\mathfrak{Y}$, $\bar{\mathrm{R}}$) where $\mathfrak{Y}\neq\emptyset$, and $\bar{\mathrm{R}}$ is an equivalence relation.

L be closed and $convex\ _{subset}$ of Æ,(CƆ)

$\mathfrak{T}$: L → L, and $\mathfrak{Q}$ : L → L,$\mathfrak{F}$: L → L satisfying the following conditions

(4.3.3.(i)) $\mathfrak{T}^2 = \mathrm{I}$, $\mathfrak{Q}^2 = \mathrm{I}$, $\mathfrak{F}^2 = \mathrm{I}$ where I is $Identity\ _{mapping}$

(4.3.3.(ii )) $\mathfrak{T}\mathfrak{Q} = \mathfrak{Q}\mathfrak{T}$, $\mathfrak{F}\mathfrak{T} = \mathfrak{T}\mathfrak{F}$, $\mathfrak{F}\mathfrak{Q} = \mathfrak{Q}\mathfrak{F}$

(4.3.3.(iii)) $\left|\left|\mathfrak{T}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}$

$$\leq \varsigma \frac{\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}}{1+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}}$$

$$+\beta\frac{\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{R}}\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{R}}+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{R}}\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{R}}}{1+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{R}}\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{R}}+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{R}}\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{R}}}$$

$$+\sigma\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{F}\mathfrak{Q}(\mathfrak{N})\right|\right|_{\bar{R}}$$

*For all*

$$\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{R}}\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{R}}+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{R}}\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{R}}\neq 0$$

$$\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{R}}+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{R}}+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{R}}+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{R}}\neq 0$$

$\forall\, \mathfrak{M},\mathfrak{N} \underbrace{\in_{\text{Å}_{\mathbb{S}}}} Æ,(\text{C}\mathfrak{O}),0\leq \varsigma,\beta,\ \sigma,\ \ \mathfrak{M}\neq\mathfrak{N}$, then $\mathfrak{T},\mathfrak{Q}\ and\ \mathfrak{F}$ have common $rough_{fixed\ point}$ if $8\varsigma+7\beta+4\sigma<8$, *also if,* $2\beta+\sigma<2$ *then* $\mathfrak{T},\mathfrak{Q}\ and\ \mathfrak{F}$ have unique $rough_{fixed\ point}$

**Corollary4.3.4** : Let (Æ,(CƆ), $\left|\left|.\right|\right|_{\bar{R}}$) be a $Rough_{Banach\ space}$ in $Approximation\ _{space}$ ( $\mathfrak{Y}$, $\bar{R}$) where $\mathfrak{Y}\neq\emptyset$, and $\bar{R}$ is an equivalence relation.

L be closed and $convex\ _{subset}$ of Æ,(CƆ)

$\mathfrak{T}$: L → L, and $\mathfrak{Q}$ : L → L,$\mathfrak{F}$: L → L satisfying the following conditions

(4.3.3.(i)) $\mathfrak{T}^2$ = I, $\mathfrak{Q}^2$ = I, $\mathfrak{F}^2$ = I where I is $Identity\ _{mapping}$

(4.3.3.(ii )) $\mathfrak{T}\mathfrak{Q}=\mathfrak{Q}\mathfrak{T}$, $\mathfrak{F}\mathfrak{T}=\mathfrak{T}\mathfrak{F}$, $\mathfrak{F}\mathfrak{Q}=\mathfrak{Q}\mathfrak{F}$

(4.3.3.(iii)) $$\left|\left|\mathfrak{T}(\mathfrak{M})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{R}}\leq\varsigma\max\left\{\begin{matrix}\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{F}\mathfrak{Q}(\mathfrak{N})\right|\right|_{\bar{R}}\\ \left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{R}}\\ \left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{R}},\\ \frac{\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{R}}\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{R}}}{1+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{F}\mathfrak{Q}(\mathfrak{N})\right|\right|_{\bar{R}}}\end{matrix}\right\}$$

$$+\beta\left[\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M})-\mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{R}}+\left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N})-\mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{R}}\right]$$

$$+ \gamma \left[ \left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M}) - \mathfrak{T}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}} + \left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{N}) - \mathfrak{T}(\mathfrak{M})\right|\right|_{\bar{\mathrm{R}}} \right]$$

$$+ \sigma \left|\left|\mathfrak{F}\mathfrak{Q}(\mathfrak{M}) - \mathfrak{F}\mathfrak{Q}(\mathfrak{N})\right|\right|_{\bar{\mathrm{R}}}$$

*For all*

$\forall\, \mathfrak{M}, \mathfrak{N} \underbrace{\in_{\text{Å}_{\mathbb{S}}}} \text{Æ}, (\text{C}\mathfrak{O}), 0 \leq \varsigma, \mathfrak{b}, \ \gamma, \ \sigma, \ \ \mathfrak{M} \neq \mathfrak{N}$, then $\mathfrak{T}, \mathfrak{Q}$ *and* $\mathfrak{F}$ have common $rough_{fixed\ point}$ if $4\varsigma + 3\,\mathfrak{b} + 3\,\gamma + \sigma < 2$, *also if,* $\mathfrak{b} + 2\gamma + \ \sigma < 1$ *then* $\mathfrak{T}, \mathfrak{Q}$ *and* $\mathfrak{F}$ have unique $rough_{fixed\ point}$

**Applications of related topics are discussed in chapter five:** In the present chapter some theoretical results are established which are very useful in engineering as well as in medical sciences. Results are also useful to overcome uncertainty. Some of applications are disused in brief. Further this chapter is subdivided five parts. Main Results are discussed in part first, third and fourth , in part second some future scopes are discussed in short and the fifth part of this chapter is given for another applications ( patent publications)

**Digital soft $\boldsymbol{\alpha - \psi}$ −contractive type mapping in digital soft metric spaces:**

**Theorem 5.1.1**: Let $(\hat{\mathrm{W}}, \tilde{E}, \widetilde{\eth_{S_{om}}}, \tilde{\rho})$ be a $complete_{\ digital\ soft\ metric}$ space. Let $(\mathfrak{F}, \varphi)$ be a $digital_{\ soft_{\ \tilde{s}\tilde{\alpha}-\tilde{\psi}-\text{contractiv}\ \ mappings}}$ from $(\hat{\mathrm{W}}, \tilde{E}, \widetilde{\eth_{S_{om}}}, \tilde{\rho})$ into itself satisfying the following:

(i) $(\mathfrak{F}, \varphi)$ is a $digital_{\ soft_{\ \tilde{\alpha}-\text{admissible}}}$

(ii) $\exists\, \widetilde{\mathcal{U}_{e0}}^{0}$, such that $\left(\tilde{\alpha}, \tilde{\emptyset}\right)\left(\widetilde{\mathcal{U}_{e0}}^{0}, (\mathfrak{F}, \varphi)\widetilde{\mathcal{U}_{e0}}^{0}\right) \tilde{\geq} \tilde{1}$,

(iii) $(\mathfrak{F}, \varphi)$ is $digital_{soft_{continuous}}$

$\Rightarrow$ $(\mathfrak{F}, \varphi)$ has a unique $soft_{fixed_{point}}$ that is, there exists $\widetilde{\mathcal{U}_{e0}}^{0} \tilde{\in} DSC(\tilde{E})$ such that $(\mathfrak{F}, \varphi)\widetilde{\mathcal{U}_{e0}}^{0} = \widetilde{\mathcal{U}_{e0}}^{0}$

**Theorem 5.1.2.** :Let $(\tilde{E}, \widetilde{\eth_{S_{om}}}, \tilde{\rho})$ be a $complete_{\ digital\ soft\ metric}$ space. Let $(\mathfrak{F}, \varphi)$ be a $digital_{\ soft_{\ \tilde{\alpha}-\tilde{\psi}-\text{contractive}\ mappings}}$ from $(\tilde{E}, \widetilde{\eth_{S_{om}}}, \tilde{\rho})$ into itself :

(i) $(\mathfrak{F}, \varphi)$ is a $digital_{\ soft_{\ \tilde{\alpha}-\text{admissibl}}}$

(ii) $\exists\, \widetilde{\mathcal{U}_{e0}}^{0} \tilde{\in} DSC(\tilde{E})$: $\left(\tilde{\alpha}, \tilde{\emptyset}\right)\left(\widetilde{\mathcal{U}_{e0}}^{0}, (\mathfrak{F}, \varphi)\widetilde{\mathcal{U}_{e0}}^{0}\right) \tilde{\geq} \tilde{1}$,

(iii) If $\{\widetilde{\mathcal{U}_{e_n}}^{n}\}$ is a sequence in $\left(\tilde{E}, \widetilde{\eth_{S_{om}}}\right)$

such that $(\tilde{\alpha}, \tilde{\emptyset})\left(\widetilde{\mathcal{U}_{e_n}}^{n}, \widetilde{\mathcal{U}_{e_{n+1}}}^{n+1}\right) \tilde{\geq} \tilde{1} \; \forall \; n \in \mathbb{N}$.

And $\widetilde{\mathcal{U}_{e_n}}^{n} \to \widetilde{\mathcal{U}_e} \; as \; n \to +\infty$, then $(\tilde{\alpha}, \tilde{\emptyset})(\widetilde{\mathcal{U}_{e_n}}^{n}, \widetilde{\mathcal{U}_e}) \tilde{\geq} \tilde{1} \; \forall \; n \in \mathbb{N}$. then $(\mathfrak{F}, \varphi)$ has a unique $fixed \; _{\cdot point \cdot}$.

## 5.3 **Advanced Manufacturing Process for Automation in Examination System using Artificial Intelligence with Soft Fixed-Point Property**

**5.3.1:** The record of answer books and attendance (RABA) in any examination system is an inexplicable but essential part of the educational institutions to mark the presence and trace the unique number of answer sheet of a particular candidate. The manually writing in the RABA of students in a conventional method is time-consuming, cumbersome to handle, and also there are chances of manual errors. To overcome these consequences, an Artificial Intelligence (AI) based system with face recognition and fingerprint based biometric system of digital RABA based on fixed point property with discission making theory is proposed. In this part an automated system is presented for face recognition and fingerprint authentication through biometrics and thereafter the answer sheet barcode is scanned for their RABA entry using a camera module. Finally, all these records are maintained over the database so that it can be fetched anytime by the institute or by the university. This system offers fast and more specific method to avoid any proxy attendance. For the said purpose some parameters are taken with fixed point properties and concepts of soft sets is used for decision . The conventional method of attendance consumes time, needs manual work and most importantly associated with the fact that the attendance can be manipulated. Also, there are chances that students who didn't respond during the attendance call but claiming for attendance later on. It is also very time consuming for the proctors to roam around the campus and checking every classroom during examination time. So, there arises a need for an efficient and easy method of checking the attendance of students during the examination. AI technology can provide a solution to this problem by application of biometrics-based attendance checking [63,115].

## 5.4 An Application of Graph mining using evidence based fuzzy soft sets

This part gives a brief idea of fuzzy soft set approaches available for grade the node(s) of relational graph. The chapter also utilized two newly suggested Ranking method to grade the nodes of the relational graph.

The Dempster-Shafer theory of evidence is a very hot way to deal with uncertainty in the information technology field. Existing Measures to determine the position of a node within a graph are either based on the graph's local structure or global structure. Each method has its own limitations. There was some scope for study to determine the approach which can utilize both factors, either local structure or global structure, to find the position of a node in a given graph. The novel method proposed here with uncertainty measure along with the evidence is named "weighted evidence combination rules" and is used to determine the credibility of each node in a given graph. Here, the modified credibility degree value is used to identify the rank of each node. The proposed approach utilizes the local or internal structure of the node with centrality calculation for initial matrix generation and the rest of the steps are focused on the global or external structure of the graph for rank determination. The result was found to be effective in determining the rank of each node in a given graph.

**5.5. Another Applications of fixed-point theory concerning with soft sets and rough sets:**

- **To diagnosis some critical diseases, like cancer, blood sugar, high blood pressure using fixed point property with soft set and rough set properties**
- **To discuss about stock marketing using Banach contraction principle and using soft set properties for decision of investment.**
- **Solution of initial value problems and boundary value problems.**
- **For speech reorganization.**
- **Network Security and Artificial intelligence.**
- To destabilise the terrorist network using fuzzy soft set method. ( For more details Published paper on the topic "A Fuzzy Soft Set Based Novel Method to Destabilize the Terrorist Network". Journal of Intelligent & Fuzzy Systems,) can be viewed with annexure
- **An image processing system using fixed point and fuzzy logic unit (Published Patent)**

The present invention discloses an image processing system using fixed point and fuzzy logic unit. The system includes, but not limited to, one or more processors and microcontrollers connected in a computer network with a centralized located processing unit is configured to evaluate fixed point calculation to image compression, image processing and flood filling of a region.

Further, the image compression is to reduce redundant image information in the digital image using the fixed-point calculation with the fuzzy logic unit, and 2D Gaussian Filtrations.